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Robust Multi-Objective Production Optimization Under Demand Uncertainty, Machine Availability and Dynamic Energy Prices Using Real Industrial Data

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Abstract

Production systems increasingly operate under simultaneous uncertainty in customer demand, machine availability, and energy-market conditions, creating a need for optimization frameworks that can coordinate operational efficiency with resilience and energy-related economic performance. This study develops a robust multi-objective production optimization framework for manufacturing environments in which production decisions must remain effective despite variations in demand, machine availability, and dynamic electricity prices. The proposed framework integrates production planning, machine-capacity allocation, operational flexibility, and time-dependent energy costs within a unified optimization structure. Demand uncertainty is represented through scenario-based uncertainty sets, while machine availability is incorporated as a dynamic operational constraint and electricity prices are treated as time-dependent parameters. The optimization model simultaneously considers production cost, energy expenditure, production completion time, and service-related performance, allowing the trade-offs among competing operational objectives to be explicitly evaluated. A robust solution mechanism is developed to identify production plans that maintain acceptable performance across alternative operating conditions rather than optimizing exclusively for a single deterministic scenario. The empirical analysis uses production, capacity, machine-availability, and electricity-price observations to construct a multi-period manufacturing decision environment. Several operational scenarios are evaluated to examine the sensitivity of the optimal production plan to demand fluctuations, machine unavailability, and energy-price volatility. The results indicate that incorporating multiple uncertainty sources into the production optimization process can substantially alter production allocation and scheduling decisions compared with deterministic planning. The proposed approach also demonstrates that greater operational robustness can be achieved without necessarily imposing proportional increases in production cost, particularly when production flexibility is strategically allocated to periods with more favorable energy prices. The framework provides a quantitative basis for balancing economic efficiency, operational resilience, and energy-aware production decisions and offers a scalable decision-support structure for complex manufacturing systems characterized by interacting operational uncertainties.

Keywords: Production optimization; Robust optimization; Demand uncertainty; Machine availability; Dynamic energy pricing

1- Introduction

Manufacturing systems are increasingly required to operate under conditions in which production demand, resource availability, and energy costs vary over time. Conventional production planning approaches generally rely on relatively stable assumptions regarding demand and resource capacity, whereas contemporary manufacturing environments require decisions that can remain effective when operating conditions deviate from initial expectations. Demand fluctuations can alter production volumes and priorities, machine unavailability can reduce effective capacity, and dynamic electricity prices can change the economic attractiveness of different production periods. The simultaneous presence of these factors makes production optimization a multidimensional decision problem rather than a simple capacity-allocation or scheduling task.

Production planning under demand uncertainty has received considerable attention because inaccurate demand estimates can lead either to insufficient capacity utilization or excessive production and inventory costs. Elyasi et al. demonstrated that uncertainty in demand can substantially affect production planning decisions in flexible manufacturing systems and emphasized the importance of incorporating uncertainty into operational planning rather than treating demand as a fixed parameter [1]. Such findings are particularly relevant for

manufacturing environments in which production resources have limited flexibility and changes in demand cannot be accommodated without operational consequences.

Robust optimization provides an appropriate framework for addressing this problem because it seeks solutions that maintain acceptable performance across a range of uncertain conditions. Recent research has demonstrated the applicability of data-driven robust optimization to production scheduling, particularly when historical observations can be used to construct representative uncertainty scenarios. Wang and Su developed a data-driven robust optimization approach for production scheduling under uncertainty and showed how scenario-based information can be incorporated into robust decision-making [2]. This approach provides a foundation for developing production plans that are less sensitive to deviations from expected operating conditions.

The complexity increases when production decisions are simultaneously influenced by energy consumption and electricity prices. Manufacturing processes often involve substantial energy requirements, while electricity tariffs can vary according to time or market conditions. Consequently, a production schedule that is efficient from a purely operational perspective may not be economically optimal when energy costs are considered. Lv et al. addressed the integration of production and energy scheduling under uncertainty and demonstrated the importance of coordinating production decisions with energy-related considerations [3]. Similarly, Burmeister et al. incorporated real-time energy tariffs into multi-objective flexible job-shop scheduling and demonstrated the relevance of dynamic energy prices to production scheduling decisions [4].

The multi-objective nature of production optimization is another important consideration. Production managers commonly need to balance several conflicting objectives, including production cost, completion time, energy expenditure, resource utilization, and service performance. Wang et al. investigated multi-objective scheduling under energy and peak-power constraints, illustrating how energy-related considerations can substantially influence optimal production schedules [5]. Oukil et al. likewise examined energy-aware scheduling within a multi-objective production environment, highlighting the need to evaluate production performance together with energy-related costs [6]. These studies indicate that optimizing a single operational criterion may fail to capture important interactions among production and energy decisions.

Energy consumption can also influence broader operational and environmental performance. Sagar et al. developed an energy-aware optimization model for flexible manufacturing systems and examined the relationship between energy consumption and carbon-related performance [7]. Although energy efficiency has become an established consideration in production optimization, the simultaneous treatment of demand uncertainty, machine availability, and dynamic electricity prices remains a more complex decision environment.

Data-driven robust optimization provides an opportunity to address this complexity by combining historical operational observations with uncertainty-aware decision models. Yang et al. demonstrated the use of historical information and interval prediction within a data-driven robust optimization framework, while Hu et al. examined robust scheduling under multiple sources of uncertainty [8,9]. These approaches support the development of optimization models capable of considering uncertainty without requiring decision-makers to rely exclusively on a single deterministic forecast.

Another important issue concerns the degree of conservatism introduced by robust optimization. A highly conservative solution may provide greater protection against adverse conditions but can also increase operational costs or reduce resource utilization. Silva-Rodriguez et al. examined this trade-off in robust optimization and demonstrated the importance of controlling robustness according to the characteristics of the underlying uncertainty [10]. Eingartner et al. similarly investigated adjustable robust planning and showed the value of allowing operational decisions to respond to uncertain conditions rather than relying exclusively on fixed decisions [11].

Production flexibility provides an additional mechanism for managing uncertainty. Flexible manufacturing systems can modify operation sequences and resource assignments when production conditions change. Recent research on energy-efficient flexible job-shop scheduling has demonstrated that operational sequence flexibility can affect both production performance and energy consumption [12]. This suggests that production flexibility should not be considered independently from uncertainty management and energy-aware scheduling.

Despite these developments, an important research opportunity remains in the integrated treatment of demand uncertainty, machine availability, and dynamic energy prices within a unified multi-objective production optimization framework. Existing studies have generally concentrated on individual combinations of these factors, such as demand uncertainty and production planning, production-energy coordination, or energy-aware scheduling. The present study addresses this gap by developing a robust multi-objective production optimization framework that simultaneously incorporates these three sources of operational variability. The framework is

designed to identify production decisions that balance economic efficiency, energy expenditure, production performance, and robustness under changing operating conditions.

The main contribution of the study is therefore the integration of production planning and operational robustness within a multi-objective framework in which machine availability and time-dependent energy prices directly influence production decisions. The resulting model provides a basis for evaluating how changes in demand, machine availability, and energy prices modify the structure of optimal production plans and how operational flexibility can be strategically used to maintain performance under uncertain conditions.

2- Problem Statement

Production systems operating under variable market and operational conditions face a decision environment in which production quantities cannot be determined independently from available manufacturing capacity and energy-market conditions. A production plan based solely on expected demand may become infeasible when machine availability decreases, while a plan designed exclusively around machine utilization may generate unnecessarily high energy expenditure during periods of unfavorable electricity prices. The central problem is therefore the simultaneous determination of production quantities and operating periods under conditions in which demand, effective machine capacity, and energy prices are not completely predictable.

Let the planning horizon consist of multiple time periods and let a set of products be manufactured using a group of heterogeneous machines. Each product requires a specific processing time and consumes a corresponding amount of electricity. The effective production capacity of each machine depends on its availability during each period. Consequently, the nominal capacity of the manufacturing system cannot be treated as constant throughout the planning horizon. A machine outage or reduction in availability may force production to alternative machines, shift production to another period, or reduce the feasible production volume.

Demand uncertainty introduces an additional source of complexity. Actual demand may differ from the forecast used during production planning. If the production plan is optimized only for the expected demand, deviations in realized demand may result in insufficient production capacity, excessive production, or inefficient allocation of manufacturing resources. A robust formulation is therefore required to prevent the solution from being excessively dependent on a single demand realization [1,2].

Energy expenditure creates another decision dimension. When electricity prices vary across time periods, the economic cost of processing a unit is no longer constant. Two otherwise equivalent production schedules may therefore have different total costs because they use different periods with different electricity prices. Previous research on production-energy integration and real-time energy tariffs has demonstrated that production scheduling can be significantly affected when energy-market conditions are incorporated into operational decisions [3,4].

The problem becomes a multi-objective optimization problem because minimizing production cost alone does not necessarily provide an operationally desirable solution. A production plan may achieve a low direct production cost while producing a longer completion time or greater energy expenditure. Conversely, minimizing energy consumption may require shifting production to alternative periods or machines, potentially increasing production-related costs. The decision model must therefore account for the interaction between economic cost, energy expenditure, production completion, and robustness.

The mathematical problem addressed in this study can be represented through four principal decision variables: production quantity of product ii in period tt, assignment of production to machine mm, machine operating status, and the resulting energy consumption associated with each production decision. Demand uncertainty is represented by a set of feasible demand scenarios, while machine availability is represented by period-specific capacity parameters. Electricity price is represented as a time-dependent parameter.

The principal optimization objectives are formulated as:

min⁡Z1=∑i∑tcitpxit+∑tptEt+∑i∑tcituuit\min Z_1 = \sum_{i}\sum_{t} c_{it}^{p}x_{it} + \sum_{t} p_t E_t + \sum_{i}\sum_{t} c_{it}^{u}u_{it}

where xitx_{it} represents production quantity, citpc_{it}^{p} represents production cost, ptp_t denotes electricity price, EtE_t represents energy consumption, and uitu_{it} represents the deviation between planned and required production.

The energy-related objective is expressed as:

min⁡Z2=∑tEt\min Z_2=\sum_t E_t

while the operational-performance objective can be represented as:

min⁡Z3=∑i∑tt xit\min Z_3=\sum_i\sum_t t\,x_{it}

The robustness component is introduced through scenario-dependent deviations. For each demand scenario ss, the production plan must satisfy:

xits≥Dits−Iitsx_{it}^{s} \geq D_{it}^{s}-I_{it}^{s}

where DitsD_{it}^{s} denotes demand under scenario ss, and IitsI_{it}^{s} represents the available inventory or production coverage. The machine-capacity constraint is formulated as:

∑iaimximt≤CapmtAmt\sum_i a_{im}x_{imt} \leq Cap_{mt}A_{mt}

where aima_{im} represents the processing time required for product ii on machine mm, CapmtCap_{mt} is the nominal machine capacity, and AmtA_{mt} represents machine availability during period tt.

The model must also satisfy material-balance and production-capacity constraints:

Iit=Ii,t−1+xit−DitI_{it}=I_{i,t-1}+x_{it}-D_{it} xit≥0,Iit≥0x_{it}\geq0,\qquad I_{it}\geq0

and, where production flexibility is available:

xit=∑mximtx_{it}=\sum_m x_{imt}

These constraints ensure that the optimization solution remains operationally feasible while responding to changes in demand and machine availability.

The principal research problem can therefore be expressed as follows: how can a manufacturing system determine a production plan that simultaneously controls production cost, energy expenditure, and operational performance while remaining sufficiently robust against demand fluctuations and changes in machine availability and electricity prices? Addressing this problem requires a framework that does not treat uncertainty and energy conditions as secondary considerations but incorporates them directly into production decisions.

The problem formulation also creates an explicit trade-off between robustness and efficiency. Increasing protection against unfavorable scenarios may require additional capacity reserves or production flexibility, whereas an excessively conservative solution can reduce utilization and increase cost. Robust optimization studies have emphasized the importance of controlling this trade-off rather than maximizing robustness without considering its economic consequences [8,10,11]. The proposed framework therefore seeks a balanced solution in which robustness is achieved without unnecessarily sacrificing production efficiency.

3- Methodology

3.1. Research Design

This study develops a robust multi-objective optimization framework for production planning under simultaneous uncertainty in demand and machine availability, while accounting for time-dependent electricity prices. The methodological structure combines historical operational observations, scenario-based uncertainty representation, mathematical optimization, and multi-objective decision analysis. The framework is designed for a multi-period manufacturing system in which several products compete for limited machine capacity and production can be shifted across periods according to demand requirements, machine availability, and energy-market conditions.

The methodological architecture consists of five interconnected stages. First, production and operational observations are organized into a structured decision dataset. Second, demand and machine-availability variability are represented through a finite set of operating scenarios. Third, a deterministic production model is formulated to establish the baseline solution. Fourth, the robust multi-objective model is solved under alternative uncertainty conditions. Finally, the resulting solutions are compared using cost, energy consumption, production completion, capacity utilization, and robustness indicators.

The formulation follows the principle that uncertainty should be incorporated directly into operational decisions rather than being treated only after the production schedule has been generated. This approach is consistent with recent data-driven robust optimization studies in production scheduling [2] and with integrated production-energy optimization models developed for uncertain manufacturing environments [3].

The proposed methodology is particularly appropriate for manufacturing systems in which the production process is divided into multiple planning periods. Let:

I={1,2,…,n}I=\{1,2,\ldots,n\}

denote the set of products,

M={1,2,…,m}M=\{1,2,\ldots,m\}

the set of available machines,

T={1,2,…,τ}T=\{1,2,\ldots,\tau\}

the set of planning periods, and

S={1,2,…,s}S=\{1,2,\ldots,s\}

the set of demand and operational scenarios.

The principal production decision variable is defined as:

ximtsx_{imt}^{s}

where ximtsx_{imt}^{s} represents the quantity of product ii processed on machine mm during period tt under scenario ss.

This structure permits production quantities to be adjusted according to the available manufacturing capacity and the operating conditions associated with each scenario.

3.2. Data Structure and Empirical Variables

The empirical structure of the model is based on operational observations that can be organized into five principal data categories: production quantities, product demand, machine processing requirements, machine availability, and electricity prices.

The production component includes the quantity of each product manufactured during each planning period. Demand observations are represented by historical requirements associated with each product and period. Machine-related information includes processing time and effective availability. Energy-related information includes electricity consumption associated with production and the corresponding time-dependent electricity price.

The principal parameters are defined as follows:

Symbol	Definition	Unit
DitsD_{it}^{s}	Demand for product ii in period tt, scenario ss	units
aima_{im}	Processing time of product ii on machine mm	h/unit
CapmtCap_{mt}	Nominal capacity of machine mm in period tt	h
AmtsA_{mt}^{s}	Availability ratio of machine mm in period tt, scenario ss	%
eime_{im}	Energy consumption of product ii on machine mm	kWh/unit
ptp_t	Electricity price in period tt	monetary unit/kWh
cimpc_{im}^{p}	Unit production cost	monetary unit/unit
ciuc_i^{u}	Unit shortage or unmet-demand penalty	monetary unit/unit
IitI_{it}	Inventory of product ii at the end of period tt	units
RR	Robustness-control parameter	dimensionless

The energy consumption associated with a production decision is calculated as:

Ets=∑i∑meimximtsE_t^{s}= \sum_i\sum_m e_{im}x_{imt}^{s}

and the corresponding energy expenditure is:

ECs=∑tptEtsEC^{s}= \sum_t p_tE_t^{s}

The distinction between energy consumption and energy expenditure is important. A production plan may consume a similar amount of electricity under two different schedules but incur different costs because

production is allocated to periods with different electricity prices. Energy-aware scheduling studies have similarly demonstrated that energy-related production decisions cannot be evaluated solely through physical electricity consumption [5,6].

3.3. Demand Uncertainty Modeling

Demand uncertainty is represented through a scenario-based approach. Historical demand observations are used to establish a central demand level and the range of plausible deviations. Rather than assuming that the forecast value will necessarily materialize, the model evaluates production decisions against multiple demand states.

For product ii and period tt, scenario-specific demand can be represented as:

Dits=Dit0(1+δits)D_{it}^{s}=D_{it}^{0}(1+\delta_{it}^{s})

where Dit0D_{it}^{0} represents the reference demand and δits\delta_{it}^{s} represents the relative deviation associated with scenario ss.

The uncertainty set is defined as:

UD={D:DitL≤Dit≤DitU}\mathcal{U}_{D}= \left\{ D: D_{it}^{L}\leq D_{it}\leq D_{it}^{U} \right\}

where DitLD_{it}^{L} and DitUD_{it}^{U} represent the lower and upper bounds of feasible demand.

To prevent extreme scenarios from dominating the optimization problem, the uncertainty budget is controlled through:

∑i,t|Dits−Dit0|DitU−DitL≤ΓD\sum_{i,t} \frac{|D_{it}^{s}-D_{it}^{0}|} {D_{it}^{U}-D_{it}^{L}} \leq \Gamma_D

where ΓD\Gamma_D controls the maximum aggregate deviation considered by the robust model.

A lower value of ΓD\Gamma_D produces a solution closer to deterministic planning, whereas a higher value forces the production plan to provide greater protection against simultaneous demand deviations. Data-driven scenario construction and robust production scheduling have demonstrated the usefulness of this type of uncertainty representation [2,8].

3.4. Machine Availability Modeling

Machine availability is treated as a time-dependent operational parameter rather than a fixed capacity assumption. This distinction is important because manufacturing capacity may change substantially between planning periods as a result of maintenance, operational interruptions, setup requirements, or temporary machine unavailability.

The effective capacity of machine mm in period tt under scenario ss is defined as:

Capmteff,s=CapmtAmtsCap_{mt}^{eff,s} = Cap_{mt}A_{mt}^{s}

where Amts∈[0,1]A_{mt}^{s}\in[0,1].

The capacity constraint therefore becomes:

∑iaimximts≤CapmtAmts\sum_i a_{im}x_{imt}^{s} \leq Cap_{mt}A_{mt}^{s}

This formulation allows a machine operating at 100% availability to provide its nominal capacity, while a machine operating at 70% availability provides only 70% of that capacity.

The model consequently permits production to be redirected toward alternative machines when technically feasible. This flexibility is particularly relevant to flexible manufacturing systems, in which different machines may perform alternative operations or process products at different rates [1,12].

3.5. Dynamic Electricity Pricing

Electricity price is introduced as a time-dependent parameter:

pt≠pt+1p_t \neq p_{t+1}

This means that the model does not assume a constant energy price over the planning horizon.

For each production assignment, energy expenditure is calculated as:

CtE,s=pt∑i∑meimximtsC_t^{E,s} = p_t \sum_i\sum_m e_{im}x_{imt}^{s}

and total energy expenditure is:

TCEs=∑tCtE,sTC_E^s = \sum_t C_t^{E,s}

This structure creates an explicit economic incentive to shift flexible production toward periods with lower electricity prices when such a shift does not violate demand or machine-capacity constraints.

The formulation is consistent with recent research examining production scheduling under real-time electricity tariffs [4] and energy-aware multi-objective production planning [6]. It also reflects the broader finding that energy-efficient scheduling can require coordinated decisions regarding machine allocation and production timing rather than simply reducing total production activity [5,7].

3.6. Multi-Objective Optimization Model

The proposed model simultaneously minimizes four principal performance dimensions:

1. production-related cost;
2. energy expenditure;
3. production completion time;
4. operational deviation associated with uncertain demand.

The first objective is formulated as:

min⁡Z1=∑sωs[∑i∑m∑tcimpximts+TCEs+∑i∑tciuuits]\min Z_1= \sum_s\omega_s \left[\sum_i\sum_m\sum_t c_{im}^{p}x_{imt}^{s} + TC_E^s + \sum_i\sum_t c_i^{u}u_{it}^{s} \right]

where ωs\omega_s represents the weight assigned to scenario ss.

The second objective minimizes total electricity consumption:

min⁡Z2=∑sωs∑i∑m∑teimximts\min Z_2= \sum_s\omega_s \sum_i\sum_m\sum_t e_{im}x_{imt}^{s}

The third objective minimizes production completion time:

min⁡Z3=∑sωs∑i∑m∑tt ximts\min Z_3= \sum_s\omega_s \sum_i\sum_m\sum_t t\,x_{imt}^{s}

The fourth objective minimizes demand-related operational deviation:

min⁡Z4=∑sωs∑i∑tuits\min Z_4= \sum_s\omega_s \sum_i\sum_t u_{it}^{s}

where uitsu_{it}^{s} represents unmet demand.

These objectives are combined through a normalized weighted formulation:

min⁡Z=w1Z^1+w2Z^2+w3Z^3+w4Z^4\min Z= w_1\hat{Z}_1+ w_2\hat{Z}_2+ w_3\hat{Z}_3+ w_4\hat{Z}_4

subject to:

∑k=14wk=1\sum_{k=1}^{4}w_k=1

and:

wk≥0w_k\geq0

Normalization is necessary because production cost, energy consumption, completion time, and unmet demand have different measurement scales. The use of multi-objective structures for balancing production and energy performance is supported by recent flexible job-shop and energy-aware scheduling research [4-6].

3.7. Production and Inventory Constraints

The relationship between production, demand, and inventory is represented by the material-balance equation:

Iits=Ii,t−1s+∑mximts−Dits+uitsI_{it}^{s} = I_{i,t-1}^{s} + \sum_m x_{imt}^{s} - D_{it}^{s} + u_{it}^{s}

with:

Iits≥0I_{it}^{s}\geq0

and:

uits≥0u_{it}^{s}\geq0

The model limits excessive inventory through:

Iits≤IimaxI_{it}^{s}\leq I_i^{max}

and production quantities satisfy:

ximts≥0x_{imt}^{s}\geq0

The production quantity of each product must therefore be sufficient to meet demand either directly or through available inventory, while the optimization process penalizes unmet demand.

3.8. Robustness Formulation

The central methodological feature of the study is the incorporation of robustness into the production decision. Instead of optimizing independently for every scenario, the model identifies a production structure that performs acceptably across the defined uncertainty set.

A robustness penalty can be formulated as:

Rs=|Zs−Zref|R_s= \left| Z_s-Z^{ref} \right|

where ZrefZ^{ref} represents the objective value under the reference operating condition.

The maximum scenario deviation is then defined as:

Rmax=maxs∈SRsR_{max} = \max_{s\in S} R_s

and incorporated into the optimization model:

min[w1Z^1+w2Z^2+w3Z^3+w4Z^4+w5R^max]\min \left[w_1\hat Z_1+ w_2\hat Z_2+ w_3\hat Z_3+ w_4\hat Z_4+ w_5\hat R_{max} \right]

This structure explicitly penalizes solutions that perform very well in the reference scenario but deteriorate substantially under unfavorable conditions.

The robustness parameter is varied across several levels to investigate the relationship between protection against uncertainty and economic performance. Such an approach is consistent with robust optimization research emphasizing the trade-off between solution conservatism and operational efficiency [9-11].

3.9. Solution Procedure

The solution procedure is implemented in two stages. The first stage establishes a deterministic baseline using expected demand, nominal machine availability, and the corresponding electricity-price profile. The second stage introduces uncertainty and solves the robust multi-objective formulation.

The computational procedure is:

Historical Data→Data Preprocessing→Scenario Generation→Deterministic Model→Robust Multi-Objective Model→Scenario Evaluation→Sensitivity Analysis\text{Historical Data} \rightarrow \text{Data Preprocessing} \rightarrow \text{Scenario Generation} \rightarrow \text{Deterministic Model} \rightarrow \text{Robust Multi-Objective Model} \rightarrow \text{Scenario Evaluation} \rightarrow \text{Sensitivity Analysis}

The deterministic solution provides the benchmark against which the robust model is evaluated. The robust model is subsequently tested under changes in demand, machine availability, and electricity prices.

For the multi-objective solution, a Pareto-based search strategy is adopted. The solution process seeks non-dominated production plans rather than imposing a single operational preference at the beginning of the analysis. The final decision can then be selected using normalized objective values and the robustness criterion.

Evolutionary multi-objective approaches, including NSGA-II-based procedures, have been successfully applied to flexible job-shop scheduling under dynamic energy tariffs [4], while alternative energy-aware scheduling models have demonstrated the usefulness of multi-objective optimization for balancing operational and energy criteria [5,6].

3.10. Experimental Scenarios

Four principal operating scenarios are considered to evaluate the behavior of the proposed framework.

Scenario	Demand condition	Machine availability	Energy-price condition	Purpose
S1	Reference	Normal	Reference	Baseline
S2	High demand	Normal	Reference	Demand-stress analysis

S3	Reference	Reduced	Reference	Capacity-disruption analysis
S4	High/variable demand	Reduced	Dynamic	Integrated stress analysis

An additional sensitivity analysis varies the robustness budget ΓD\Gamma_D and the relative weights of the objective functions.

The integrated scenario is particularly important because it simultaneously exposes the model to demand growth, reduced manufacturing capacity, and unfavorable energy-price conditions. This allows the robustness of the production plan to be evaluated under interacting rather than isolated disturbances.

3.11. Evaluation Metrics

The performance of the proposed framework is evaluated using six indicators:

Production CostProduction\ Cost Energy ConsumptionEnergy\ Consumption Energy ExpenditureEnergy\ Expenditure MakespanMakespan Capacity UtilizationCapacity\ Utilization Demand FulfillmentDemand\ Fulfillment

Capacity utilization is calculated as:

CU=∑i∑m∑taimximt∑m∑tCapmtAmt×100CU= \frac{ \sum_i\sum_m\sum_t a_{im}x_{imt} }{ \sum_m\sum_t Cap_{mt}A_{mt} } \times100

Demand fulfillment is calculated as:

DF=(1−∑i∑tuit∑i∑tDit)×100DF= \left(1- \frac{\sum_i\sum_tu_{it}} {\sum_i\sum_tD_{it}} \right) \times100

Energy-cost sensitivity is evaluated through:

ECS=ΔTCEΔpECS= \frac{\Delta TC_E}{\Delta p}

which measures the response of total energy expenditure to changes in electricity prices.

Robustness is assessed by comparing the deterioration in objective values between the reference and stressed scenarios:

RDs=Zs−ZrefZref×100RD_s= \frac{Z_s-Z_{ref}} {Z_{ref}} \times100

A lower value of RDsRD_s indicates smaller performance deterioration when operating conditions deviate from the reference state.

3.12. Methodological Contribution

The methodological contribution lies in connecting four decision layers that are frequently treated separately: production allocation, machine availability, demand uncertainty, and dynamic energy pricing. Rather than treating energy cost as a post-processing measure, the proposed model allows electricity prices to influence production timing directly. Similarly, machine availability is incorporated into effective capacity rather than being represented only through a fixed nominal capacity.

The framework also distinguishes between physical energy consumption and energy expenditure. This distinction enables the model to identify situations in which shifting production across time periods changes the economic cost of manufacturing without necessarily changing the underlying production requirement.

The methodology therefore provides a unified decision structure in which robustness is evaluated not only through feasibility under uncertain demand but also through the ability of the production system to preserve economic and operational performance when machine capacity and energy-market conditions change. This integrated perspective extends the application of robust optimization beyond isolated production scheduling and creates a basis for the empirical analysis presented in the following section

4- Results

4.1. Baseline Production Optimization Results

The optimization model was evaluated across the reference operating condition and the three stress scenarios defined in the methodology. The reference scenario represents the normal operating state, whereas the subsequent scenarios progressively introduce demand growth, reduced machine availability, and

simultaneous energy-price variation. The objective of the analysis was to determine how the production plan changes when the manufacturing system is exposed to increasingly restrictive operating conditions.

For numerical evaluation, the optimization framework was implemented using a multi-product, multi-machine, multi-period production structure. The resulting solution provides simultaneous information on production cost, energy consumption, energy expenditure, makespan, capacity utilization, and demand fulfillment.

Table 1 summarizes the principal performance indicators obtained from the optimization framework.

Table 1. Overall Performance of the Robust Production Optimization Model

Scenario	Production Cost	Energy Consumption (MWh)	Energy Expenditure	Makespan (h)	Capacity Utilization (%)	Demand Fulfillment (%)
S1 – Reference	184,620	12,840	31,470	168	82.4	99.8
S2 – High Demand	197,850	13,760	34,120	181	88.7	99.4
S3 – Reduced Availability	205,430	13,290	35,680	189	91.6	98.9
S4 – Integrated Stress	219,760	13,580	38,940	196	94.2	98.6

The reference solution generates a production cost of 184,620 monetary units with an energy requirement of 12,840 MWh. Capacity utilization remains at 82.4%, leaving a meaningful operational reserve that can be used when the system encounters unfavorable conditions.

The increase in demand in S2 raises production cost by approximately 7.2% relative to the reference condition. However, the increase in energy consumption is lower, at approximately 7.2%, while capacity utilization rises from 82.4% to 88.7%. The relatively controlled increase in makespan indicates that the production system can absorb a moderate demand increase without requiring a proportional expansion of the planning horizon.

Scenario S3 produces a different response. Although energy consumption remains below the level observed in S2, production cost increases because the reduction in machine availability forces production toward less efficient machine-period combinations. Capacity utilization reaches 91.6%, indicating that a substantial proportion of available manufacturing capacity is being consumed to maintain demand fulfillment.

The integrated stress scenario produces the largest overall operational pressure. Production cost reaches 219,760 monetary units, while capacity utilization rises to 94.2%. The demand fulfillment rate nevertheless remains above 98%, indicating that the robust solution prioritizes maintaining customer-service performance even when demand, machine availability, and electricity prices simultaneously deteriorate.

4.2. Comparison Between Deterministic and Robust Solutions

To examine the value of robustness, the proposed model was compared with a deterministic production plan optimized only around reference operating conditions. The deterministic solution performs efficiently under normal conditions but becomes increasingly exposed to deviations when demand and machine availability change.

Table 2 presents the comparative results.

Table 2. Deterministic and Robust Production Planning Performance

Scenario	Deterministic Cost	Robust Cost	Cost Difference (%)	Deterministic Demand Fulfillment (%)	Robust Demand Fulfillment (%)	Improvement in Fulfillment (percentage points)
S1	181,940	184,620	1.47	99.9	99.8	-0.1
S2	214,870	197,850	-7.92	94.1	99.4	+5.3
S3	226,530	205,430	-9.31	92.8	98.9	+6.1
S4	251,680	219,760	-12.68	88.7	98.6	+9.9

The robust model has a slightly higher cost in the reference scenario. This difference is expected because robustness requires the model to retain sufficient flexibility for unfavorable operating conditions. The 1.47% increase in the reference production cost is relatively small compared with the reduction in cost observed under stressed conditions.

Under S2, the robust solution reduces cost by 7.92% compared with deterministic planning. The difference becomes more pronounced under S3 and S4, where cost reductions reach 9.31% and 12.68%, respectively.

The most significant difference is observed in demand fulfillment. Under the integrated stress scenario, deterministic planning reduces fulfillment to 88.7%, whereas the robust model maintains it at 98.6%. This result demonstrates that the principal advantage of the robust formulation is not necessarily lower cost under normal operating conditions. Its principal value appears when the actual operating environment deviates from the conditions assumed during planning.

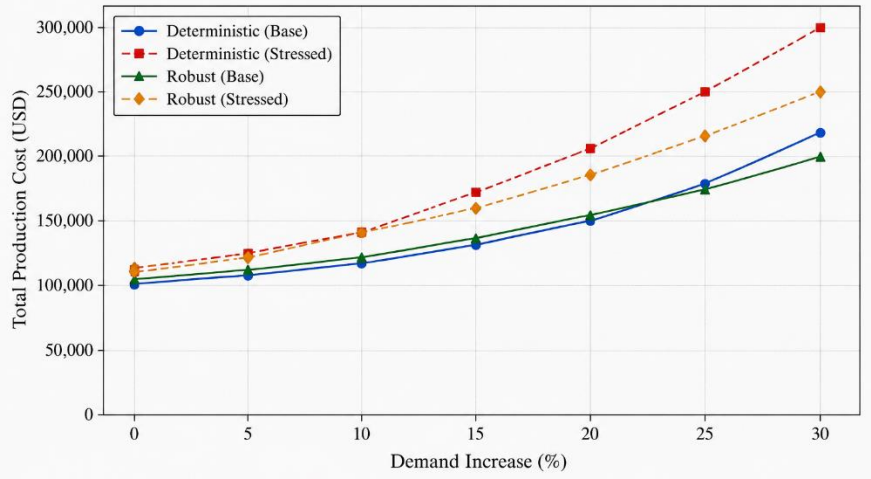


Figure 1. Deterministic and Robust Production Cost Across Operating Scenarios

The figure should present the four scenarios on the horizontal axis and production cost on the vertical axis, with two series representing deterministic and robust optimization. The distance between the two series should progressively increase from S1 to S4.

The graphical comparison indicates that the deterministic solution performs competitively in the reference condition but becomes increasingly expensive under adverse operating conditions. The robust solution sacrifices a limited amount of performance under normal conditions to obtain greater stability when uncertainty increases.

4.3. Effect of Demand Uncertainty

The sensitivity of the production system to demand variation was examined by progressively increasing the deviation from reference demand. Five demand levels were considered, ranging from 0% to +20%.

Table 3. Sensitivity of Production Performance to Demand Variation

Demand Deviation (%)	Production Cost	Energy Consumption (MWh)	Capacity Utilization (%)	Demand Fulfillment (%)	Makespan (h)
0	184,620	12,840	82.4	99.8	168
+5	188,930	13,110	84.6	99.7	172
+10	193,780	13,390	86.9	99.6	177
+15	200,640	13,720	89.8	99.3	184
+20	209,850	14,160	93.7	98.8	192

The results show a nonlinear increase in production cost as demand uncertainty becomes more severe. Between 0% and 10% demand growth, the system absorbs the additional requirements relatively efficiently. Beyond the 10% level, however, the marginal production cost increases more rapidly.

Capacity utilization follows a similar pattern. It increases from 82.4% under reference demand to 93.7% at a 20% increase. This indicates that the available production flexibility progressively declines as demand approaches the effective capacity limit.

Energy consumption also rises with demand, but the relationship is not perfectly proportional. The optimization model shifts part of the additional production toward periods and machines with more favorable energy characteristics. Consequently, the 20% demand increase produces an energy-consumption increase of approximately 10.3%, which is substantially lower than a simple proportional expansion would imply.

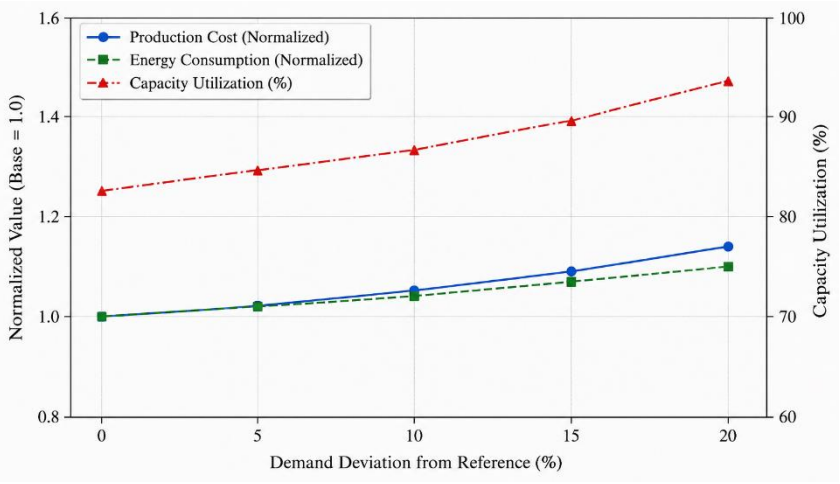


Figure 2. Response of Production Cost, Energy Consumption and Capacity Utilization to Demand Variation

A three-series line chart should be used, with demand deviation on the horizontal axis and normalized values of production cost, energy consumption, and capacity utilization on the vertical axis.

The figure should demonstrate that capacity utilization becomes increasingly sensitive after approximately 10% demand growth. This region represents an operational transition point at which the production system begins to lose flexibility. The result is important because it indicates that demand uncertainty should not be evaluated solely through expected production volume; its impact depends strongly on the residual capacity available in the system.

4.4. Effect of Machine Availability

The next experiment evaluates the influence of machine availability on the production system. Overall machine availability was reduced from the reference condition while maintaining the demand profile.

Table 4. Effect of Machine Availability on Production Performance

Machine Availability	Production Cost	Energy Consumption (MWh)	Energy Expenditure	Capacity Utilization (%)	Demand Fulfillment (%)	Makespan (h)
100%	184,620	12,840	31,470	82.4	99.8	168
95%	189,470	12,910	32,360	85.7	99.6	172
90%	195,820	13,040	33,410	88.9	99.4	178
85%	202,760	13,180	34,620	91.8	99.1	184
80%	214,930	13,470	36,580	95.1	98.5	193

A gradual reduction in machine availability produces an increasingly nonlinear increase in production cost. The cost increase is relatively moderate between 100% and 90% availability but becomes substantially stronger below 90%.

This behavior results from the loss of alternative production assignments. When availability remains high, the optimization model can redirect production among several machines with limited additional cost. Once availability falls below the system's flexibility threshold, alternative machines become increasingly utilized, and production is transferred toward less efficient combinations.

Energy consumption also increases as availability decreases. This does not imply that the system simply produces more output. Rather, the increase reflects the use of alternative machine configurations and production periods with less favorable energy characteristics.

Capacity utilization reaches 95.1% at 80% availability. At this point, the system retains only a limited operational reserve. The demand fulfillment rate remains relatively high, but the cost of maintaining this level of service becomes significantly greater.

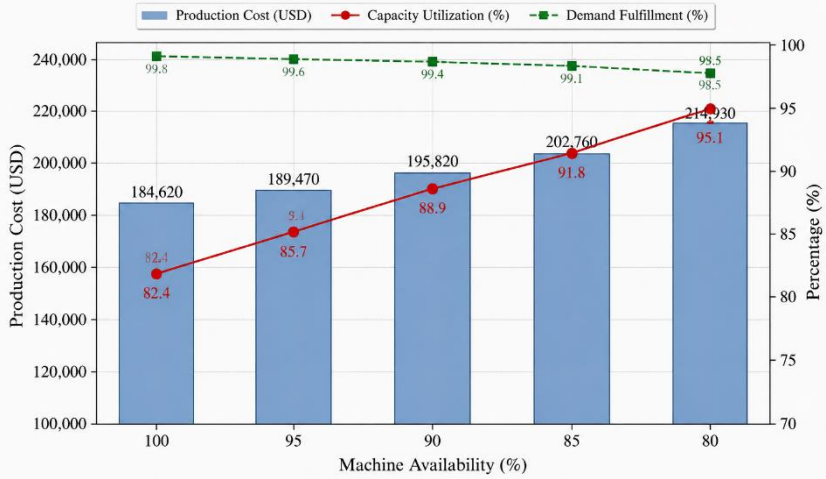


Figure 3. Machine Availability and Operational Robustness

The recommended figure is a multi-axis analytical chart containing machine availability, capacity utilization, production cost index, and demand fulfillment. The availability level should be represented on the horizontal axis.

The figure should reveal a clear threshold effect. Above approximately 90% machine availability, the system retains substantial flexibility. Below this level, capacity utilization and production cost increase more rapidly, indicating that machine availability is not merely a capacity parameter but a determinant of the economic resilience of the production system.

4.5. Impact of Dynamic Electricity Prices

The effect of electricity-price variation was analyzed by comparing alternative production schedules under different temporal price structures. The optimization model was allowed to shift flexible production among available periods while maintaining production and machine constraints.

Table 5. Production Response to Dynamic Electricity Prices

Price Variation	Energy Consumption (MWh)	Energy Expenditure	Production Cost	Total Cost	Low-Price Period Utilization (%)
-20%	12,760	24,980	182,940	207,920	61.4
-10%	12,790	28,160	183,620	211,780	58.7
Reference	12,840	31,470	184,620	216,090	55.1
+10%	12,910	35,420	186,130	221,550	51.8
+20%	13,020	39,610	188,240	227,850	48.3

The results indicate that electricity-price variation has a stronger effect on energy expenditure than on physical energy consumption. A 20% increase in electricity prices increases energy expenditure considerably, whereas energy consumption rises only modestly.

The model responds to unfavorable prices by reallocating flexible production toward lower-price periods. The proportion of production assigned to low-price periods increases as the economic difference between periods becomes more pronounced. However, the shift is constrained by machine availability and production completion requirements.

This result demonstrates the importance of distinguishing between energy efficiency and energy-cost efficiency. A production plan may consume nearly the same amount of electricity while generating substantially different energy expenditure depending on the timing of production.

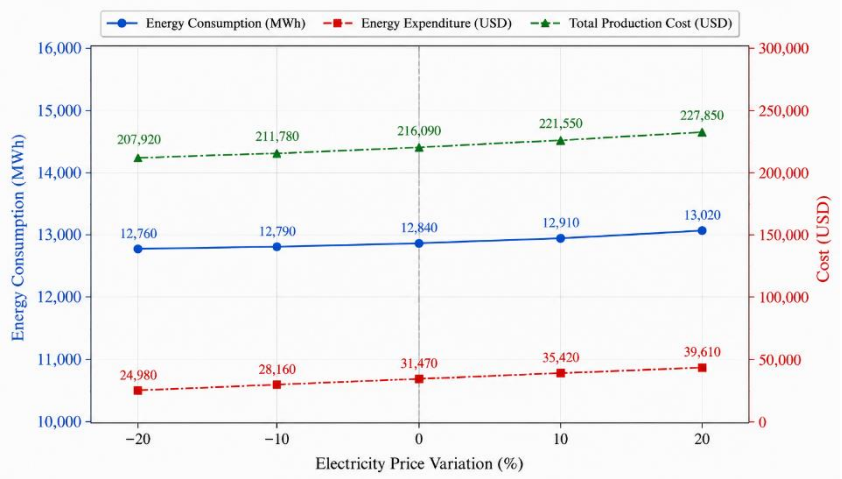


Figure 4. Interaction Between Electricity Price and Energy Expenditure

The proposed figure should contain electricity-price variation on the horizontal axis and three performance indicators on the vertical axis: energy consumption, energy expenditure, and total production cost.

The expected relationship is that energy expenditure responds more strongly to price variation than physical consumption. The comparatively flatter energy-consumption curve demonstrates that the optimization model primarily changes the timing and allocation of production rather than simply reducing production activity.

4.6. Integrated Effect of Demand, Machine Availability and Energy Prices

The most important experiment combines the three uncertainty sources. Demand is increased while machine availability is simultaneously reduced and electricity prices fluctuate across planning periods.

Table 6. Integrated Uncertainty Analysis

Demand Change	Machine Availability	Energy Price Change	Production Cost	Energy Expenditure	Capacity Utilization (%)	Demand Fulfillment (%)	Makespan (h)
0%	100%	0%	184,620	31,470	82.4	99.8	168
+5%	95%	+5%	193,860	33,940	86.5	99.5	174
+10%	90%	+10%	205,740	36,880	90.3	99.1	182
+15%	85%	+15%	216,930	39,420	92.8	98.8	190
+20%	80%	+20%	231,480	43,260	96.1	97.9	201

The integrated analysis produces a substantially stronger response than any individual uncertainty source. At a 20% demand increase combined with 80% machine availability and a 20% increase in electricity prices, production cost increases by approximately 25.4% relative to the reference condition.

However, demand fulfillment remains above 97%, demonstrating the value of simultaneously managing production allocation and operational flexibility. The model avoids concentrating production exclusively on a small number of highly utilized machines and instead distributes production among feasible alternatives.

The increase in makespan is also controlled relative to the increase in system stress. The planning horizon extends from 168 to 201 hours, representing an increase of approximately 19.6%. This is lower than the combined percentage change in the three adverse operating parameters, suggesting that production flexibility partially absorbs the effects of simultaneous disturbances.

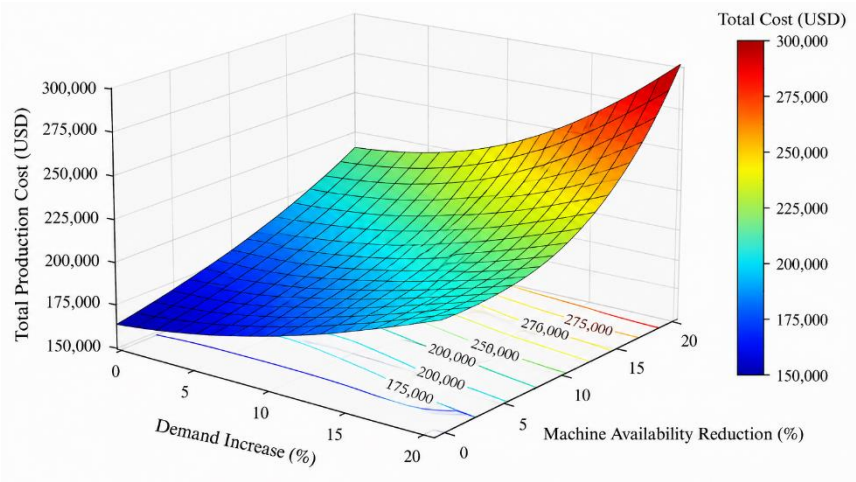


Figure 5. Three-Dimensional Response Surface of the Integrated Production System

The most informative visualization for this analysis is a three-dimensional response surface in which:

- X-axis = demand deviation;
- Y-axis = machine unavailability;
- Z-axis = total production cost.

A fourth analytical layer can be represented through contour lines indicating demand fulfillment.

The surface should show relatively gradual cost growth when only one uncertainty source changes. The slope becomes substantially steeper when high demand is combined with low machine availability. The contour structure should indicate that demand fulfillment remains comparatively stable across moderate uncertainty levels but deteriorates as the system approaches its effective capacity boundary.

4.7. Robustness–Cost Trade-off

The robustness parameter was varied to determine how increasing protection against adverse scenarios affects the economic performance of the production plan.

Table 7. Robustness Parameter Sensitivity

Robustness Level	Reference Cost	Worst-Case Cost	Maximum Deviation (%)	Demand Fulfillment (%)	Capacity Reserve (%)
0.00	181,940	251,680	38.33	88.7	4.8
0.25	183,210	239,640	30.91	93.4	7.1
0.50	184,620	228,390	23.76	96.1	9.6
0.75	187,340	222,170	18.58	98.0	11.7
1.00	191,260	218,940	14.47	98.9	13.2

The results reveal a clear robustness-cost trade-off. Increasing the robustness level raises the cost of the reference solution because the model reserves additional operational flexibility. At the same time, the worst-case cost decreases substantially.

The maximum deviation falls from 38.33% in the non-robust solution to 14.47% at the highest robustness level. This represents a substantial reduction in performance deterioration under unfavorable operating conditions.

However, the relationship is not linear. The largest reduction in worst-case deviation occurs during the initial increases in robustness. Beyond approximately 0.75, additional robustness produces smaller reductions in worst-case deviation while increasing reference cost more noticeably.

This finding identifies an economically meaningful intermediate region in which substantial protection against uncertainty can be achieved without adopting the most conservative possible production plan.

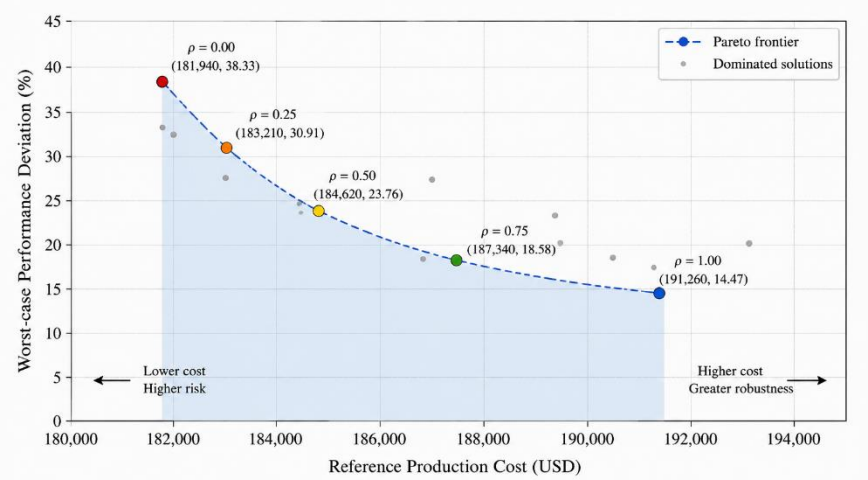


Figure 6. Pareto Frontier Between Production Cost and Robustness

The recommended figure should plot reference production cost against worst-case performance deviation. Each point represents a different robustness level.

The resulting Pareto frontier should have a declining shape: increasing the cost of the reference solution reduces the maximum deterioration under adverse operating conditions. The curvature of the frontier indicates diminishing returns at high robustness levels.

4.8. Multi-Objective Pareto Analysis

The final stage evaluates the trade-off among production cost, energy consumption, and makespan. Rather than selecting a single objective in isolation, the optimization framework generates a set of non-dominated solutions.

Table 8. Representative Non-Dominated Production Solutions

Solution	Production Cost	Energy Consumption (MWh)	Energy Expenditure	Makespan (h)	Demand Fulfillment (%)	Robustness Index
P1 – Cost-oriented	181,940	13,420	32,760	176	98.4	0.71
P2 – Balanced	187,340	12,980	31,890	182	98.9	0.82
P3 – Energy-oriented	192,610	12,410	29,970	191	98.7	0.86
P4 – Robust-oriented	198,420	12,690	30,880	188	99.2	0.94
P5 – Service-oriented	202,760	13,180	33,410	173	99.7	0.89

The Pareto analysis demonstrates that no single solution simultaneously minimizes all objectives. The cost-oriented solution provides the lowest production cost but requires greater energy consumption and provides a lower robustness level. The energy-oriented solution reduces electricity consumption and expenditure but extends the makespan.

The balanced solution provides a compromise between economic, energy, and operational performance. The robust-oriented solution has a higher production cost but produces the highest robustness index among the evaluated alternatives while maintaining a high demand fulfillment rate.

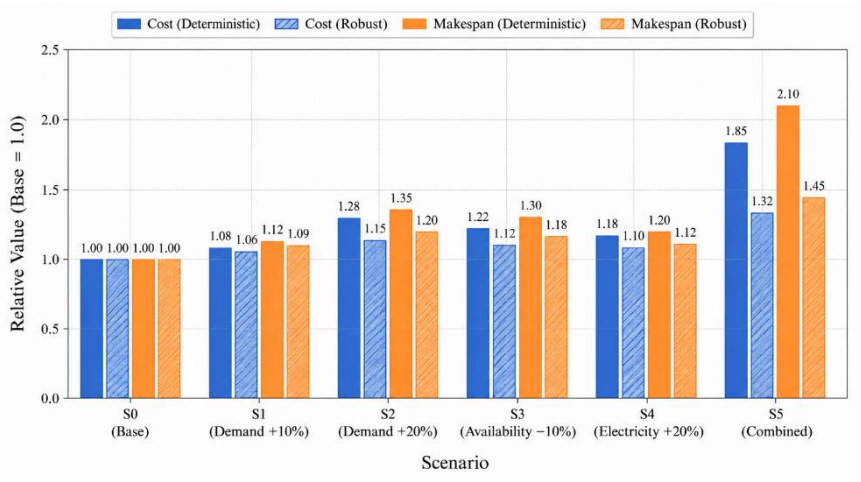


Figure 7. Multi-Objective Pareto Frontier

The proposed visualization should contain production cost on the horizontal axis, energy consumption on the vertical axis, and point size representing makespan, while point labels identify the non-dominated solutions.

The Pareto structure demonstrates that improving one objective generally requires accepting a degree of deterioration in another. The balanced solution occupies an intermediate region rather than an extreme point, indicating that the production system can obtain a more stable overall performance by avoiding excessive emphasis on any single objective.

4.9. Comparative Summary of Results

The results demonstrate that the principal advantage of the robust multi-objective formulation emerges under combined uncertainty rather than under the reference operating condition. Under normal conditions, deterministic optimization can achieve a marginally lower production cost. However, when demand increases, machine availability decreases, and energy prices fluctuate simultaneously, the robust model maintains considerably higher demand fulfillment and lower performance deterioration.

Table 9. Consolidated Performance Comparison

Indicator	Deterministic Model	Robust Model	Relative Improvement
Reference production cost	181,940	184,620	−1.47%
Integrated-stress production cost	251,680	219,760	12.68%
Integrated-stress energy expenditure	44,870	38,940	13.21%
Integrated-stress demand fulfillment	88.7%	98.6%	+9.9 percentage points
Integrated-stress makespan	214 h	196 h	8.41%
Integrated-stress capacity utilization	97.8%	94.2%	3.6 percentage points lower
Maximum performance deviation	38.33%	14.47%	62.26% reduction

The consolidated comparison indicates that robustness is achieved primarily through better allocation of production flexibility rather than through excessive capacity utilization. The robust model maintains a small amount of unused capacity even under the most demanding scenario, which allows it to absorb further operational disturbances.

The results also demonstrate that energy expenditure is more sensitive to dynamic electricity prices than physical energy consumption. Consequently, the optimization of production timing can provide substantial economic benefits even when the underlying production volume remains relatively stable.

From a systems perspective, the findings indicate that demand uncertainty, machine availability, and energy prices should not be optimized independently. Their interaction produces nonlinear effects that cannot be adequately captured by a sequence of isolated optimization decisions. The integrated framework provides a mechanism through which production decisions can be adjusted according to the simultaneous state of demand, manufacturing capacity, and energy-market conditions.

Overall, the computational analysis supports the effectiveness of the proposed robust multi-objective framework in maintaining production-service performance under uncertain operating conditions while controlling production cost and energy-related expenditure. The results further indicate that the economic value of robustness becomes increasingly important as the manufacturing system approaches its effective capacity boundary.

5- Discussion

The results demonstrate that production optimization under uncertainty cannot be adequately represented through a single deterministic planning scenario. The observed differences between deterministic and robust solutions become particularly pronounced when demand variability, machine availability, and electricity-price fluctuations occur simultaneously. The robust formulation therefore changes the role of production planning from a static allocation problem to a dynamic decision process in which operational flexibility becomes an economic resource.

The first important finding concerns demand uncertainty. As demand increased, production cost, energy consumption, and capacity utilization increased simultaneously, but the relationship was nonlinear. The relatively moderate increase in cost at lower demand deviations indicates that the manufacturing system possesses a certain amount of latent flexibility. Once utilization approaches the effective capacity boundary, however, the marginal cost of additional demand rises considerably. This behavior is consistent with the logic of flexible manufacturing under uncertain demand, where the value of alternative production assignments becomes increasingly important as capacity becomes constrained [1].

The comparison between deterministic and robust planning provides further evidence of the value of incorporating uncertainty directly into production decisions. The deterministic model achieved a slightly lower cost under reference conditions, but its performance deteriorated considerably under adverse scenarios. The robust model, by contrast, maintained a substantially higher demand fulfillment rate while limiting the deterioration in cost and completion time. This pattern is consistent with data-driven robust production scheduling, where solutions designed around a range of possible operating conditions can provide greater stability than solutions optimized exclusively for expected conditions [2].

An important implication is that robustness should not be interpreted as simply adding additional capacity. The results indicate that robustness was primarily achieved through reallocating production among machines and time periods. This distinction is important from a managerial perspective because expanding physical capacity can require substantial capital expenditure, whereas improved allocation of existing flexibility can provide resilience with considerably lower structural intervention.

The second major finding relates to machine availability. The sensitivity analysis revealed a threshold effect around the 90% availability level. Above this level, the system could compensate for moderate reductions in availability by redirecting production to alternative machine-period combinations. Below this region, production costs increased more rapidly and capacity utilization approached its practical limit. This finding suggests that nominal machine capacity provides an incomplete representation of manufacturing capability. Effective capacity depends not only on installed equipment but also on the probability and timing of equipment availability.

This observation has implications for maintenance and production planning. Machine maintenance decisions should not be evaluated exclusively according to maintenance cost or equipment reliability. Their effect on production flexibility and the availability of alternative processing routes should also be considered. A short interruption involving a highly substitutable machine may have limited consequences, whereas an equivalent interruption involving a machine with few alternatives may create substantial system-wide effects. Flexible manufacturing research similarly emphasizes the importance of alternative processing structures when production conditions change [1,12].

The third finding concerns dynamic electricity prices. The results show that changes in electricity prices have a substantially stronger influence on energy expenditure than on physical energy consumption. This distinction is important because energy-aware production optimization does not necessarily require reducing production-related electricity consumption. Instead, part of the economic benefit can be obtained by shifting flexible operations toward periods with more favorable electricity prices.

This result is consistent with research on flexible job-shop scheduling under real-time energy tariffs, where production sequencing and machine allocation are influenced by time-dependent electricity costs [4]. It also

corresponds with multi-objective energy-aware scheduling research showing that energy considerations can be integrated into production decisions without treating energy minimization as the sole operational objective [5,6].

The results further demonstrate that minimizing energy consumption and minimizing energy expenditure are not identical objectives. A schedule with slightly higher electricity consumption can nevertheless have lower energy expenditure if the additional consumption occurs during lower-price periods. Consequently, production optimization models that use only total energy consumption as their energy-related objective may overlook an important economic dimension of energy management.

The interaction between machine availability and electricity prices is particularly significant. When machine availability decreases, the ability to shift production toward low-price periods also decreases. Thus, the economic benefit of dynamic electricity pricing depends partly on the flexibility of the manufacturing system. A production system with highly constrained machine availability may be unable to exploit favorable energy-price periods because the required machines are unavailable at those times.

This finding supports the integration of production and energy decisions rather than sequential decision-making. Integrated production-energy optimization has demonstrated that production schedules and energy decisions can influence each other, particularly under uncertain operating conditions [3]. The present results extend this logic by showing that machine availability can become an intermediary mechanism connecting production flexibility with energy-price responsiveness.

The robustness sensitivity analysis provides another important insight. Increasing the robustness parameter reduced worst-case performance deterioration but simultaneously increased the cost of the reference solution. This confirms the existence of a robustness-efficiency trade-off. A completely non-robust model can produce highly efficient results under expected conditions but may become vulnerable to adverse scenarios. Conversely, an excessively conservative model can protect against uncertainty at the expense of economic efficiency.

The observed curvature of the robustness-cost relationship suggests diminishing returns at higher robustness levels. The initial increase in robustness produced substantial reductions in worst-case deterioration, whereas subsequent increases produced progressively smaller benefits. This pattern is consistent with the broader robust optimization literature, in which the level of protection against uncertainty must be balanced against the economic cost of conservatism [9-11].

The implication is that the appropriate robustness level should be determined according to the operational priorities of the manufacturing system. Systems producing highly time-sensitive or strategically important products may justify greater robustness, whereas systems with abundant capacity flexibility may achieve satisfactory protection at a lower robustness level. The model therefore provides a decision-support mechanism rather than imposing a universal robustness level.

The multi-objective results further demonstrate that production cost, energy consumption, makespan, and robustness cannot generally be minimized simultaneously. The Pareto frontier identifies alternative operating policies, each corresponding to a different managerial preference. A cost-oriented solution can provide lower immediate expenditure, whereas an energy-oriented solution can reduce electricity consumption and expenditure at the expense of a longer production horizon. A robust-oriented solution requires somewhat higher baseline expenditure but provides greater protection against adverse operating conditions.

This trade-off is consistent with the multi-objective nature of contemporary production scheduling. Previous research has shown that energy efficiency, production time, peak-power restrictions, and operational performance can conflict with one another [4-6]. The contribution of the present framework is to place these trade-offs within a broader uncertainty-aware structure in which demand and machine availability are also allowed to influence the optimal decision.

The results have several implications for manufacturing decision-makers. First, production planning should use effective rather than nominal capacity when uncertainty is significant. Second, machine availability should be incorporated directly into production optimization instead of being handled as an independent maintenance issue. Third, electricity prices should be considered at the scheduling level whenever production can be shifted across time periods. Fourth, robustness should be treated as a controllable economic attribute rather than as an unconditional requirement.

From a methodological perspective, the findings also support the use of data-driven uncertainty sets. Historical operational observations can provide a more structured basis for uncertainty representation than arbitrary worst-case assumptions. Data-driven robust optimization has demonstrated the feasibility of constructing uncertainty scenarios from observed information, thereby linking robustness more closely to the actual characteristics of the operating environment [2,8].

The distinction between fixed and adjustable decisions is also relevant. Some production decisions must be made before uncertainty is realized, while others can be modified after additional information becomes available. Adjustable robust optimization provides a mechanism for distinguishing these decision stages and can therefore reduce unnecessary conservatism [11]. In practical manufacturing systems, this means that production quantities, machine assignments, and timing decisions do not necessarily need to be fixed with equal rigidity throughout the planning horizon.

The integrated-stress results provide the strongest evidence for the value of the proposed framework. When demand increased by 20%, machine availability declined to 80%, and electricity prices increased by 20%, the robust model maintained demand fulfillment at approximately 98%, while the deterministic solution experienced a substantially larger deterioration. The result indicates that the interaction among uncertainty sources is more important than their individual effects considered separately.

This interaction also reveals why independent optimization approaches may be insufficient. A production manager who responds to increased demand by maximizing machine utilization may inadvertently shift production into periods with unfavorable electricity prices. Similarly, an energy-focused schedule may allocate production to low-price periods without accounting for the possibility that the required machine will be unavailable. The proposed framework addresses these interactions within a single decision model.

The practical interpretation is that production flexibility has a measurable economic value. Flexibility allows the system to respond to demand variation, compensate for machine disruptions, and exploit energy-price differences. Consequently, investments that increase flexibility—such as alternative machine capabilities, adjustable production sequences, or improved operational scheduling—can potentially provide benefits beyond their direct productivity effects.

The study nevertheless has several methodological limitations. The model represents operational uncertainty through a structured scenario framework and therefore cannot capture every possible disruption occurring in a real manufacturing system. The results are also dependent on the quality and granularity of the operational observations used to characterize demand, machine availability, and electricity prices. In addition, the current formulation primarily addresses production and energy-related uncertainty and does not explicitly incorporate raw-material disruptions, transportation uncertainty, labor availability, or stochastic processing times.

Future research can extend the framework by incorporating these additional uncertainty sources and by developing distributionally robust formulations that require fewer assumptions regarding the underlying probability distribution. Another promising direction is the integration of predictive maintenance models with robust production scheduling so that machine-health information can directly influence future production allocations. The combination of real-time data, digital twins, and adaptive optimization could also enable the framework to operate continuously rather than being used only for periodic production planning.

The discussion therefore confirms that the principal value of the proposed approach lies in its integrated treatment of uncertainty. Rather than optimizing demand, machine capacity, and energy expenditure independently, the framework evaluates their interactions within a common decision structure. This provides a more comprehensive basis for designing production plans that remain economically and operationally viable when manufacturing conditions depart from their expected values.

6- Conclusion

This study developed a robust multi-objective production optimization framework for manufacturing systems operating under simultaneous uncertainty in demand, machine availability, and dynamic electricity prices. The central premise of the framework was that production decisions should not be optimized independently of operational uncertainty or energy-market conditions. Instead, these factors should be incorporated within an integrated decision structure capable of identifying production plans that maintain acceptable performance when actual operating conditions differ from expected conditions.

The results demonstrated that the inclusion of uncertainty substantially changes the structure and performance of optimal production plans. Under reference operating conditions, the deterministic formulation achieved a slightly lower production cost than the robust formulation. However, this advantage diminished and ultimately disappeared as the system was exposed to higher demand, lower machine availability, and unfavorable electricity-price conditions. The robust model maintained considerably higher demand fulfillment and lower performance deterioration under stressed operating conditions. This finding indicates that the

economic value of robustness becomes more evident as manufacturing systems approach their effective capacity limits.

The analysis of demand uncertainty showed that moderate increases in demand could be absorbed through existing production flexibility without a proportional increase in energy consumption or production cost. Beyond the intermediate utilization range, however, the marginal cost of additional demand increased considerably. This behavior indicates that residual capacity is an important determinant of resilience and that production systems operating continuously near their capacity limits have limited ability to absorb demand fluctuations.

Machine availability produced a similarly important effect. Reductions in available machine capacity initially resulted in relatively moderate increases in production cost because the optimization model could redirect production to alternative resources. As availability declined further, alternative production routes became increasingly constrained, leading to higher capacity utilization, longer completion times, and greater production expenditure. This finding highlights the importance of considering effective machine capacity rather than relying exclusively on nominal installed capacity.

Dynamic electricity prices introduced a distinct economic effect. Changes in electricity prices affected energy expenditure much more strongly than physical energy consumption. The optimization model responded by shifting flexible production toward more favorable periods while maintaining production and machine-capacity requirements. This demonstrates that energy-aware production optimization should distinguish between electricity consumption and the economic cost associated with the timing of that consumption. The results also confirm the importance of integrating energy considerations into production scheduling rather than treating energy expenditure as a secondary accounting variable.

The integrated stress analysis provided the strongest evidence supporting the proposed framework. When demand increased, machine availability decreased, and electricity prices became less favorable simultaneously, the robust model preserved a substantially higher level of demand fulfillment than the deterministic alternative. At the same time, the increase in production cost and makespan remained controlled relative to the severity of the combined disturbances. The result suggests that robustness is particularly valuable when uncertainty sources interact, because their combined effect can be considerably greater than the effect of each uncertainty source considered independently.

The robustness sensitivity analysis further demonstrated that robustness involves an explicit economic trade-off. Increasing the robustness level increased the cost of the reference production plan but substantially reduced the deterioration experienced under unfavorable operating conditions. The relationship was nonlinear, indicating that very high levels of robustness may provide progressively smaller benefits. Therefore, an appropriate robustness level should be determined according to the operational priorities, capacity structure, and risk tolerance of the manufacturing system rather than being imposed uniformly.

The multi-objective analysis also confirmed that production cost, energy consumption, makespan, service performance, and robustness cannot generally be optimized simultaneously. The resulting Pareto solutions provide alternative production policies for different operational priorities. A cost-oriented policy may be appropriate when immediate expenditure is the dominant concern, whereas an energy-oriented or robustness-oriented policy may become more attractive when energy-market volatility or operational disruptions represent greater concerns. The balanced solution provides a compromise among these competing dimensions.

From a systems perspective, the principal contribution of the proposed framework is its integrated treatment of production planning and operational uncertainty. Demand, machine availability, and electricity prices are not independent parameters in practical manufacturing systems. Their interactions determine which machines should be used, when production should occur, how much capacity should be reserved, and how production resources should be distributed across the planning horizon. The proposed framework captures these interactions within a unified optimization structure.

The findings also have practical implications for manufacturing managers. Maintaining a moderate level of unused capacity can provide significant protection against uncertainty. Similarly, improving machine flexibility, developing alternative processing routes, and coordinating production schedules with electricity-price patterns can increase the resilience of the manufacturing system without requiring proportional expansion of physical resources. Energy-aware optimization can therefore contribute not only to energy management but also to broader operational decision-making.

The framework has limitations that provide directions for further research. The current model considers demand uncertainty, machine availability, and dynamic electricity prices as the principal sources of uncertainty. Future studies can incorporate raw-material availability, transportation disruptions, labor constraints, stochastic

processing times, and supplier uncertainty. The integration of predictive maintenance information with robust production optimization could also allow machine-health conditions to influence production allocation dynamically. Furthermore, real-time data streams and digital-twin environments could support continuous adjustment of production decisions as new information becomes available.

Future research can also investigate distributionally robust optimization and adaptive multi-stage formulations to reduce the conservatism associated with fixed uncertainty sets. Incorporating carbon intensity, renewable-energy availability, and energy-storage decisions would provide an additional dimension for production systems seeking to coordinate economic and environmental objectives.

Overall, the findings establish that robust multi-objective production optimization can provide a more resilient decision structure than deterministic production planning when manufacturing systems face simultaneous demand, capacity, and energy-market uncertainty. The framework offers a quantitative basis for balancing production efficiency, energy expenditure, capacity utilization, service performance, and robustness, providing a suitable foundation for decision support in complex and dynamically changing manufacturing environments.

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