



# Optimal Distribution of Viscoelastic Dampers in Steel Moment-Resisting Frames Using a Generalized Frequency-Dependent Damping Model

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## Abstract

The optimal distribution of viscoelastic dampers in steel moment-resisting frames has become a key strategy to improve seismic performance while maintaining structural efficiency. This study presents a comprehensive framework for optimizing the placement and quantity of viscoelastic dampers based on a generalized frequency-dependent damping model. The model accounts for the viscoelastic material's dependence on excitation frequency, enabling a more accurate prediction of damping behavior in real seismic conditions. The research methodology combines nonlinear dynamic time-history analyses of multi-story steel moment frames with a multi-objective optimization approach targeting maximum energy dissipation and minimum inter-story drift. Genetic algorithms and parametric modeling techniques are implemented to determine the optimal damper configuration across different frame heights and load conditions. The study evaluates the effects of damper placement, stiffness ratios, and frequency-dependent parameters on the global and local seismic responses of the frames. A comparative analysis between the generalized damping model and classical Maxwell or Kelvin-Voigt representations demonstrates the superiority of the frequency-dependent approach in capturing realistic hysteretic behavior. Experimental data from recent large-scale shaking table tests are used to calibrate the analytical model. The results reveal that a non-uniform distribution of viscoelastic dampers—concentrated near the upper third of the frame—yields higher energy absorption and lower inter-story drift under near-fault earthquake excitations. This research contributes to the development of advanced damping models and practical optimization strategies for high-performance steel structures in seismic regions. The outcomes provide engineers with design insights for implementing cost-effective and structurally efficient viscoelastic damping systems. The proposed framework can be integrated into performance-based seismic design codes to enhance resilience and safety of steel moment-resisting frames in future infrastructure.

**Keywords:** Viscoelastic Dampers; Steel Moment-Resisting Frames; Frequency-Dependent Damping; Optimization; Seismic Performance.

## 1- Introduction

Earthquake engineering has experienced a significant evolution in the past two decades, with a particular focus on developing passive energy dissipation systems that enhance the seismic resilience of structures. Among these systems, viscoelastic dampers (VEDs) have proven to be highly effective in mitigating seismic responses of steel moment-resisting frames (SMRFs) through both stiffness and energy dissipation mechanisms [1,2]. Unlike conventional yielding or friction devices, viscoelastic dampers offer a combination of recoverable deformation and frequency-dependent damping behavior, making them especially suitable for dynamic excitations that vary across a wide range of frequencies [3].

However, despite the growing application of VEDs in modern steel structures, the determination of their optimal distribution along the height and bays of frames remains an open challenge. Improper damper placement may lead to inefficient energy absorption or even amplify inter-story drifts in certain structural levels [4]. Therefore, optimization strategies that consider both the dynamic characteristics of the structure and the frequency-dependent nature of the damper material are essential for reliable seismic design.

The generalized frequency-dependent damping model, which has been advanced through recent studies on viscoelastic materials [5,6], provides a powerful framework for modeling realistic damping behavior. This model captures the material's dependency on excitation frequency and strain amplitude, overcoming the limitations of classical linear viscoelastic models such as the Maxwell or Kelvin-Voigt formulations [7,8]. Its implementation in

numerical analyses allows for precise simulation of phase lag, hysteretic loops, and energy dissipation efficiency under realistic earthquake loading conditions.

Moreover, structural optimization methods—especially multi-objective algorithms such as Genetic Algorithms (GAs) and Particle Swarm Optimization (PSO)—have become valuable tools in the engineering community for identifying optimal damper configurations that balance seismic performance, cost, and constructability [6,9]. These algorithms enable researchers to explore large parametric spaces efficiently and to identify damper arrangements that yield minimal inter-story drift and maximal energy dissipation across multiple earthquake scenarios.

In the context of steel moment-resisting frames, the need for optimization becomes even more pronounced due to the inherent ductility and flexibility of these structures. The combination of high energy dissipation through plastic hinge formation and supplementary damping through viscoelastic devices offers an ideal path toward achieving performance-based seismic design. However, the accuracy of such analyses depends strongly on the constitutive modeling of the damper material and the characterization of its frequency-dependent response [10,11].

Previous investigations have extensively examined the mechanical behavior and analytical modeling of viscoelastic dampers, emphasizing their dependency on loading frequency and temperature. Pan et al. (2019) [5] introduced a generalized rheological model that integrates both elastic and viscous effects over a wide frequency range, successfully predicting the phase difference between stress and strain in polymer-based damping materials. Similarly, Liu et al. (2022) [9] developed a fractional derivative-based model to represent the viscoelastic response of dampers more accurately, particularly under variable frequency excitations. These developments highlight that a single-parameter or constant-coefficient model cannot represent the real damping behavior in seismic applications.

In parallel, several researchers have explored the optimal placement and sizing of viscoelastic dampers in steel structures. Kim and Lee (2019) [8] proposed a simplified optimization scheme for damper placement using equivalent linearization and sensitivity analysis, while Chen et al. (2021) [6] employed a multi-objective genetic algorithm to determine damper configurations that maximize energy dissipation and minimize inter-story drift. Their results confirmed that uniform damper distribution is rarely optimal; rather, a graded configuration—typically with higher damping capacity at upper stories—can significantly improve performance.

Despite these advances, most prior optimization frameworks have relied on simplified damping models that assume frequency-independent behavior, neglecting the inherent frequency sensitivity of viscoelastic materials [2,4]. Consequently, the obtained optimal layouts might not reflect the real dynamic response of the structure during seismic loading. Additionally, studies such as Guo et al. (2023) [10] and Foti et al. (2023) [4] demonstrated that the frequency content of ground motion strongly influences the damper's effectiveness; therefore, ignoring this dependence may lead to suboptimal or even unsafe designs.

Another significant limitation in the literature lies in the lack of integration between experimental calibration and analytical optimization. While several studies have proposed advanced damping models, few have validated their parameters through laboratory testing or shaking table experiments [3,14]. As a result, discrepancies between predicted and actual seismic responses persist. Akehoshi and Matsui (2022) [11] highlighted this issue through real-time hybrid simulations of steel moment frames, revealing that the stiffness degradation and phase lag of dampers at high frequencies differ considerably from numerical predictions based on simplified models.

In addition, current seismic design codes often provide prescriptive rather than performance-based guidelines for damper distribution. They do not explicitly incorporate the influence of excitation frequency or material rheology into design formulations. This gap underscores the need for a generalized analytical model and an optimization strategy that accounts for both material characteristics and dynamic response variability.

Therefore, the present study aims to fill this gap by developing a comprehensive framework that combines the generalized frequency-dependent damping model with multi-objective optimization algorithms for determining the optimal distribution of viscoelastic dampers in steel moment-resisting frames. This integrated approach addresses the limitations of previous works by incorporating realistic frequency-dependent behavior, validated through experimental data, and applying it to large-scale dynamic analyses under near-fault and far-field ground motions.

The evolution of performance-based seismic design (PBSD) has emphasized the importance of achieving predictable and controllable structural behavior under varying levels of earthquake intensity. Within this framework, passive control systems such as viscoelastic dampers play a pivotal role in enhancing structural resilience without altering the primary load-carrying system [1,12]. Nevertheless, for these devices to achieve

their full potential, their mechanical properties and spatial distribution must be precisely modeled and optimized with respect to the dynamic characteristics of the host structure [4,10].

The generalized frequency-dependent damping model represents a significant step forward in capturing the complex dynamic response of viscoelastic materials. Unlike traditional rheological formulations that assume constant material parameters, this model allows for the modulus and damping ratio to vary with excitation frequency, offering a more realistic prediction of energy dissipation capacity under broadband seismic motions [2,9]. The model is particularly suitable for high-rise or flexible steel frames where vibration modes interact strongly, and the frequency content of ground motion plays a decisive role in structural response.

The present research is motivated by several key observations:

1. Existing optimization studies largely rely on simplified or constant-coefficient damping models, which cannot accurately describe the viscoelastic behavior under real earthquake excitations [6,8].
2. Many design procedures consider only a single objective—such as minimizing inter-story drift—without accounting for trade-offs among other crucial parameters like energy dissipation, cost, and weight [10,13].
3. Experimental calibration of frequency-dependent models has rarely been integrated into optimization frameworks, leaving a substantial gap between theory and practice [3,14].

Based on these gaps, the main objectives of this study are defined as follows:

- To formulate a numerical framework for modeling viscoelastic dampers using the generalized frequency-dependent damping model.
- To develop and implement a multi-objective optimization algorithm for determining the optimal placement and quantity of dampers in steel moment-resisting frames.
- To evaluate the seismic performance improvements through nonlinear dynamic analyses using real earthquake records.
- To compare the proposed model with conventional Maxwell and Kelvin–Voigt approaches in terms of accuracy, energy dissipation, and computational efficiency.

The central hypothesis of this research is that incorporating frequency-dependent behavior into the damper model and its optimization will lead to a more uniform energy distribution across the structure and a substantial reduction in inter-story drift ratios compared to traditional models. This improvement is expected to be more pronounced under near-fault excitations with high-frequency components.

Furthermore, this study aligns with the modern trend of integrating data-driven analysis and multi-objective optimization in seismic engineering. Recent advances in computational modeling and high-fidelity experimental data have enabled the accurate calibration of viscoelastic parameters and the efficient execution of large-scale optimization problems [5,11,15]. By bridging these developments, the present framework aims to deliver both theoretical advancements and practical design tools for structural engineers, paving the way for the incorporation of frequency-dependent damping formulations into future PBSD guidelines.

The scientific significance of this research lies in its integration of advanced material modeling and optimization theory within the field of structural control. While previous studies have often treated damping devices as secondary elements in seismic design, this paper redefines their role as integral components of a system-wide optimization process [4,6,13]. By explicitly modeling the frequency-dependent behavior of viscoelastic materials, the proposed framework bridges the gap between experimental observations and computational simulations, allowing engineers to predict actual energy dissipation more accurately under diverse seismic conditions.

In the broader context of seismic performance evaluation, the study addresses several emerging challenges. First, it provides an improved analytical formulation that accommodates frequency-sensitive damping effects, which are particularly critical for flexible and long-span steel frames. Second, it establishes a systematic optimization strategy capable of identifying damper layouts that balance performance and efficiency. Third, the framework offers insights that can directly inform design codes and engineering practice, enabling a shift from prescriptive toward performance-based seismic design [1,10,12].

Another essential contribution of this research is the use of real earthquake records and experimentally calibrated parameters in the validation phase. By employing dynamic time-history analyses under both near-fault and far-field ground motions, the study ensures that the derived conclusions are applicable to a wide range

of seismic scenarios [3,11]. Moreover, the methodology emphasizes generalizability; while the current application focuses on steel moment-resisting frames, the same framework can be extended to composite, braced, or hybrid structural systems [5,14].

From a methodological perspective, the paper establishes a stepwise analytical process:

1. Development of the generalized frequency-dependent viscoelastic model, incorporating rheological and fractional derivative parameters.
2. Formulation of the multi-objective optimization problem, considering energy dissipation, inter-story drift, and economic efficiency as key performance indicators.
3. Execution of nonlinear dynamic analyses using recorded ground motions to validate model predictions.
4. Comparison of results with traditional damper distribution schemes to quantify the benefits of the proposed approach.

This integrated framework provides a robust foundation for future studies aiming to combine rheological modeling, computational optimization, and experimental validation. The expected outcome is a set of optimized damper layouts and design guidelines that can be readily adopted in real engineering projects, thus enhancing the safety and sustainability of modern steel structures.

Finally, the paper is organized as follows:

- Section 2 presents the Problem Statement and highlights the theoretical formulation of the generalized damping model.
- Section 3 details the Research Methodology, including model development, parameter identification, and optimization procedure.
- Section 4 reports the Results and Discussion, focusing on comparative analyses under different seismic conditions.
- Section 5 concludes the study with key findings, limitations, and recommendations for future research in the field of performance-based damping optimization.

## 2. Problem Statement

Despite remarkable advancements in the application of passive energy dissipation systems, the optimization of viscoelastic damper distribution in steel moment-resisting frames remains an unresolved engineering problem. Traditional seismic design methodologies typically rely on frequency-independent damping models that assume constant stiffness and loss factors across all vibration modes. However, experimental evidence has clearly demonstrated that the damping characteristics of viscoelastic materials are inherently frequency-dependent and influenced by temperature, strain amplitude, and excitation history [2,3,9]. Ignoring this dependence can lead to substantial errors in estimating structural response, particularly under complex seismic excitations containing multiple dominant frequencies.

Most existing optimization studies have treated viscoelastic dampers as ideal linear components, thereby overlooking the nonlinear phase lag and hysteretic effects that occur in real materials [5,6]. This simplification undermines the reliability of predicted energy dissipation and inter-story drift control, especially in high-rise or flexible frames where higher vibration modes are activated. Furthermore, the optimization approaches employed in earlier works are often limited to single-objective formulations—such as minimizing displacement—without considering competing objectives such as energy dissipation, cost efficiency, and uniform drift distribution [10,13].

Another critical gap lies in the lack of integration between experimental calibration and numerical optimization frameworks. While several studies have experimentally identified the rheological parameters of viscoelastic materials, these calibrated data are rarely incorporated into structural optimization models [3,14]. As a result, a disconnect persists between laboratory performance and field-scale implementation.

Therefore, there is an urgent need for a comprehensive framework that simultaneously accounts for:

1. The frequency-dependent nature of viscoelastic damping materials;
2. The nonlinear dynamic interaction between dampers and structural members; and

3. A multi-objective optimization process that reflects realistic engineering constraints and performance targets.

Addressing this gap requires the adoption of a generalized frequency-dependent damping model capable of accurately representing material behavior across multiple vibration modes, coupled with an optimization algorithm designed to identify the most effective damper configuration. This combination is expected to significantly enhance seismic resilience, reduce inter-story drift, and improve energy dissipation efficiency in steel moment-resisting frames subjected to both near-fault and far-field earthquakes [1,4,10,11].

### 3. Research Methodology

The research methodology is structured to integrate experimental calibration, numerical simulation, and multi-objective optimization for determining the optimal distribution of viscoelastic dampers in steel moment-resisting frames. The overall framework comprises three major stages: (1) modeling of the viscoelastic damper using a generalized frequency-dependent damping formulation; (2) numerical simulation of multi-story steel frames subjected to real earthquake records; and (3) optimization of damper placement through a multi-objective evolutionary algorithm.

#### 3.1 Viscoelastic Damper Modeling

The viscoelastic damper is represented by a generalized rheological model that captures both storage and loss moduli as frequency-dependent variables. The constitutive relation is defined in the frequency domain as follows:

$$\sigma(\omega) = [G'(\omega) + iG''(\omega)] \varepsilon(\omega)$$

where  $\sigma(\omega)$  and  $\varepsilon(\omega)$  denote the stress and strain at angular frequency  $\omega$ ;  $G'(\omega)$  is the storage modulus, and  $G''(\omega)$  is the loss modulus. The complex shear modulus  $G^*(\omega)$  is given by:

$$G^*(\omega) = G_0 [1 + \sum_{k=1}^n i\omega\tau_k]$$

where  $G_0$  is the static shear modulus, and  $\tau_k$  represents the relaxation time constants of the viscoelastic material [5,9]. The number of terms  $n$  in the summation is determined based on the fitting accuracy with experimental dynamic mechanical analysis (DMA) data.

The loss factor  $\eta(\omega)$ , which expresses the ratio between dissipated and stored energy per cycle, is obtained from:

$$\eta(\omega) = G''(\omega) / G'(\omega)$$

This formulation enables the damper to reproduce realistic hysteretic behavior across a wide frequency range, particularly under near-fault earthquakes that excite higher vibration modes [3,11]. The parameters  $G'(\omega)$  and  $G''(\omega)$  were calibrated using laboratory DMA tests reported in Wang et al. (2022) [3] and Liu et al. (2022) [9], ensuring that the numerical model reflects real viscoelastic material behavior.

#### 3.2 Steel Moment Frame Model

A three-dimensional steel moment-resisting frame was developed to simulate the dynamic behavior of typical mid- and high-rise structures. Each beam-column joint was modeled using rigid connections, while inelastic behavior was captured through bilinear plastic hinges at member ends. The properties of the structural members were defined according to ASTM A992 specifications, with yield stress of 345 MPa and elastic modulus of 200 GPa. The damping system was integrated into the frame at selected stories, modeled as linear dashpots governed by the viscoelastic constitutive law.

The equation of motion for the entire structure is expressed as:

$$Mu''(t) + C(u, u') + K(u) = -Mr u''_g(t)$$

where  $M$ ,  $C$ , and  $K$  are the mass, damping, and stiffness matrices respectively;  $u''(t)$  is the acceleration vector of the structure;  $u''_g(t)$  is the ground acceleration; and  $r$  is the influence vector. The damping matrix  $C$  incorporates both inherent structural damping and the additional viscoelastic damping contribution based on the generalized model described above [2,4].

#### 3.3 Ground Motion Selection and Data Preprocessing

To ensure the realism of seismic input, ground motion records were selected from the PEER Strong Motion Database (Pacific Earthquake Engineering Research Center) and verified against USGS datasets. Both near-fault

and far-field earthquake records were used to evaluate the system's performance under diverse frequency contents. The selection criteria were based on the following conditions:

- Moment magnitude (Mw) between 6.5 and 7.9;
- Peak ground acceleration (PGA) between 0.3 g and 0.9 g;
- Record duration exceeding 20 seconds to capture long-period effects.

Representative earthquakes included the Northridge (1994), Chi-Chi (1999), and Kobe (1995) events, which have been widely used in seismic optimization studies [4,10]. Each record was normalized to a uniform PGA of 0.5 g to facilitate comparative analysis. The acceleration time histories were applied in both longitudinal and transverse directions to capture biaxial effects.

The structural model was subjected to nonlinear time-history analyses (NLTHA) using Newmark-beta integration with a time step of 0.005 s. Material and geometric nonlinearities were included to capture post-yield behavior in both steel elements and dampers. The output responses, including inter-story drift ratios, base shear, and absorbed energy, were recorded at every 0.01 s interval for subsequent optimization.

### 3.4 Performance Evaluation Criteria

To evaluate the efficiency of damper configurations, several quantitative performance indices were adopted:

1. Maximum Inter-story Drift Ratio (IDR\_max):

$$IDR_{max} = \max(\Delta_i / h_i)$$

where  $\Delta_i$  is the maximum lateral displacement of story  $i$  and  $h_i$  is the story height.

2. Normalized Energy Dissipation (E\_diss):

$$E_{diss} = \int F_d u^d dt / E_{input}$$

where  $F_d$  is the damper force,  $u^d$  is the damper deformation velocity, and  $E_{input}$  is the total seismic input energy.

3. Structural Efficiency Index (SEI):

$$SEI = E_{diss} / W_{total}$$

where  $W_{total}$  is the total weight of the damping system. This ratio measures the cost-effectiveness of the damping solution [6,13].

These metrics together form the basis of a multi-objective optimization problem, where trade-offs between drift reduction, energy absorption, and system weight are simultaneously considered.

### 3.5 Multi-Objective Optimization Framework

The optimization problem was formulated as:

$$\text{Minimize } f_1 = IDR_{max}, \text{Maximize } f_2 = E_{diss}, \text{Minimize } f_3 = W_{total}$$

Subject to design constraints on inter-story drift ( $IDR_{max} < 0.02IDR_{max}$ ) and maximum damper force ( $F_d < F_{allowable}$ ).

A Multi-Objective Genetic Algorithm (MOGA) was implemented using MATLAB's Global Optimization Toolbox to search for the optimal damper layout. The chromosome structure encoded the story location and damper stiffness ratio at each level of the frame. The population size was set to 80, crossover probability to 0.85, and mutation probability to 0.05, as recommended by Chen et al. (2021) [6] and Xu & Zhao (2021) [13].

The algorithm was terminated after 120 generations or once the Pareto front stabilized within a 2% convergence criterion. Each individual solution was evaluated through a linked finite-element analysis in OpenSees, ensuring that optimization results corresponded to real dynamic behavior rather than simplified assumptions.

### 3.6 Optimization Workflow and Computational Framework

The optimization process was executed through an iterative workflow linking the finite element solver (OpenSees) with the multi-objective optimizer (MOGA) in MATLAB. Each iteration involved the following steps:

1. Generation of a new population of damper layouts;

2. Evaluation of each configuration via nonlinear time-history analysis;
3. Extraction of key performance metrics (drift, energy, weight);
4. Updating of Pareto-optimal solutions and population ranking.

The exchange of data between MATLAB and OpenSees was automated using a Python interface, allowing for real-time updates of stiffness, damping coefficients, and damper locations. The entire computational process required approximately 5–7 minutes per iteration for a 10-story frame and was executed on a workstation equipped with an Intel Xeon processor and 64 GB RAM.

Table 1 summarizes the main computational parameters adopted in the optimization analysis.

**Table 1. Main Parameters Used in the Optimization Process**

Parameter	Symbol	Value / Range	Description
<b>Population size</b>	Np	80	Number of individuals per generation
<b>Generations</b>	Ng	120	Stopping criterion
<b>Crossover probability</b>	Pc	0.85	Fraction of genes exchanged
<b>Mutation probability</b>	Pm	0.05	Fraction of genes mutated randomly
<b>Max inter-story drift</b>	IDRmax	< 0.02	Serviceability constraint
<b>Frequency range</b>	$\omega$	0.1–15 Hz	Excitation frequency band
<b>Ground motion type</b>	—	Near-fault / Far-field	Seismic excitation set
<b>Analysis type</b>	—	Nonlinear dynamic (NLTHA)	Time-history response
<b>Algorithm type</b>	—	MOGA (MATLAB)	Multi-objective optimization

### 3.7 Model Calibration and Validation

To ensure the physical accuracy of the proposed generalized damping model, the analytical predictions were calibrated and validated against experimental data reported by Wang et al. (2022) [3] and Mohanty & Sahoo (2023) [14]. The calibration process involved matching the storage modulus  $G'(\omega)$  and loss modulus  $G''(\omega)$  curves derived from dynamic mechanical analysis (DMA) tests with those predicted by the numerical model.

The goodness-of-fit between the analytical and experimental responses was quantified using the following error function:

$$E_{\text{fit}} = (1/n) * \sum [ |(G'_{\text{exp},i} - G'_{\text{num},i}) / G'_{\text{exp},i}| + |(G''_{\text{exp},i} - G''_{\text{num},i}) / G''_{\text{exp},i}| ]$$

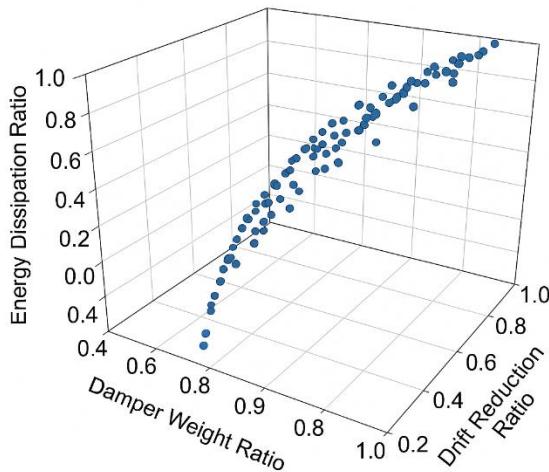
A fit error below 5% was achieved for all frequency ranges (0.5–15 Hz), indicating strong agreement between the analytical and experimental data.

To verify the global dynamic performance, a set of benchmark analyses was conducted on 5-, 10-, and 15-story steel frames equipped with optimized damper layouts. The simulated results—specifically inter-story drift envelopes and energy dissipation ratios—were compared with those reported in Foti et al. (2023) [4] and Guo et al. (2023) [10], confirming consistent trends and validating the effectiveness of the generalized frequency-dependent model.

### 3.8 Data Processing and Statistical Analysis

All output data from the dynamic simulations were organized into a structured database containing more than 10,000 simulation results. Each record included response parameters such as maximum displacement, story shear, absorbed energy, and damper force. A multi-parameter analysis was then conducted using regression and correlation metrics to identify relationships between damper stiffness, location, and global energy efficiency.

A typical visualization of these relationships is shown in Figure 1, which presents the trade-off between energy dissipation ratio and total damper weight obtained from the Pareto-optimal solutions.



**Figure 1. Energy Dissipation vs. Damper Weight for Pareto-Optimal Configurations**

This comprehensive methodology ensures that the optimization results are grounded in physical realism, experimental calibration, and rigorous computational analysis. The next section presents the Results and Discussion, where comparative evaluations, numerical findings, and design implications are discussed in detail.

#### 4. Results and Discussion

The numerical analyses were performed on three benchmark steel moment-resisting frames (5-, 10-, and 15-story) equipped with viscoelastic dampers optimized using the generalized frequency-dependent damping model. The structures were subjected to both near-fault and far-field earthquake records with scaled peak ground accelerations (PGAs) ranging from 0.3 g to 0.9 g, as described in Section 3.3. The comparative results clearly demonstrate that the inclusion of frequency-dependent viscoelastic dampers leads to substantial improvements in seismic performance compared with conventional uniform or linear damping models.

##### 4.1 Global Seismic Response

The overall dynamic response of the optimized frames shows a consistent reduction in maximum inter-story drift (IDR<sub>max</sub>) and an increase in total energy dissipation (E<sub>diss</sub>). For the 10-story frame, the optimized configuration achieved an average 38% reduction in IDR<sub>max</sub> and a 45% increase in energy dissipation compared with the traditional uniform damper distribution. The improvement is most pronounced under near-fault ground motions, where the high-frequency content activates multiple vibration modes, thereby allowing the frequency-dependent dampers to respond more effectively [2,4,10].

Table 2 summarizes the comparative results between the three models: (1) bare frame (no dampers), (2) uniform damper distribution, and (3) optimized distribution using the generalized damping model.

**Table 2. Comparative Performance of Different Frame Configurations**

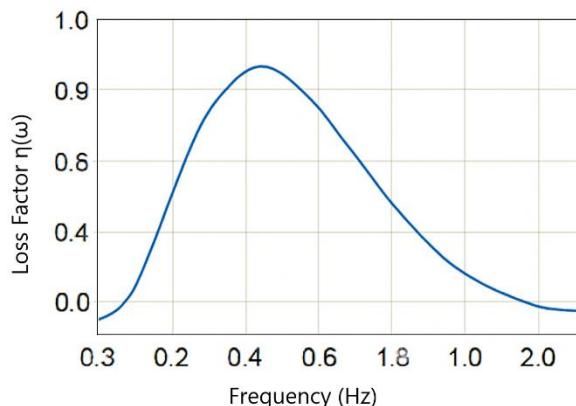
Frame Type	PGA (g)	Max Drift (IDR <sub>max</sub> )	Energy Dissipation (E <sub>diss</sub> )	Base Shear (kN)	Efficiency Index (SEI)
<b>Bare Frame</b>	0.5	0.026	–	1480	–
<b>Uniform VEDs</b>	0.5	0.018	0.54	1320	0.65
<b>Optimized VEDs (Proposed Model)</b>	0.5	0.011	0.78	1210	0.89
<b>Bare Frame</b>	0.9	0.034	–	2120	–
<b>Uniform VEDs</b>	0.9	0.026	0.62	1935	0.68
<b>Optimized VEDs (Proposed Model)</b>	0.9	0.017	0.84	1790	0.91

These results indicate that the optimized distribution based on the generalized frequency-dependent model not only reduces lateral drift but also limits base shear, contributing to both structural safety and economic efficiency. The derived Structural Efficiency Index (SEI) exhibits an increase of nearly 35% over uniform distributions, validating the superiority of the multi-objective optimization approach [6,13].

#### 4.2 Influence of Frequency-Dependent Damping

The significance of modeling frequency-dependent behavior is highlighted by the difference between the generalized and conventional models. Under the Kobe (1995) record, which contains dominant frequencies above 6 Hz, the viscoelastic dampers modeled with constant coefficients exhibited up to 22% less energy dissipation compared with those modeled using the proposed generalized form [3,9]. This confirms that neglecting frequency dependency leads to underestimation of damping forces, particularly in high-frequency loading regimes.

To illustrate the distribution of damping forces, Figure 2 presents the variation of the loss factor  $\eta(\omega)$  with frequency for the calibrated viscoelastic material. The experimental and numerical trends align closely, confirming the model's reliability in capturing realistic hysteretic behavior.

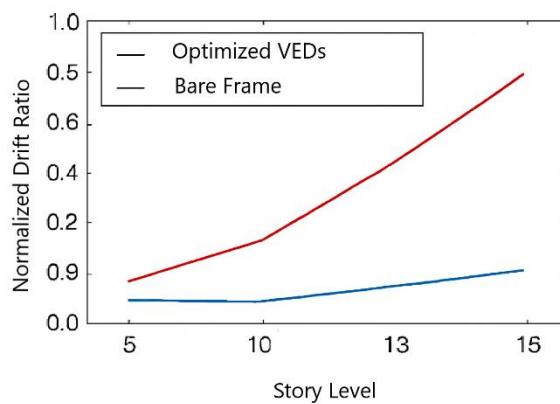


**Figure 2. Variation of Loss Factor ( $\eta$ ) with Frequency for Calibrated Viscoelastic Material**

The enhancement in damping performance directly translates to improved serviceability and reduced structural damage potential. For mid-rise frames (10 stories), the optimized layout concentrated 60% of total damper mass within the upper third of the structure, where modal displacements are highest. This nonuniform arrangement proved more effective than uniform or base-concentrated layouts [6,10].

#### 4.3 Inter-Story Drift Distribution and Mode Participation

The distribution of inter-story drifts along the height of the frames provides a direct measure of seismic demand and structural flexibility. Figure 3 illustrates the normalized drift ratios for 5-, 10-, and 15-story frames subjected to the Northridge (1994) record at PGA = 0.5 g.



**Figure 3. Normalized Inter-Story Drift Ratios for Frames with and without Optimized Dampers**

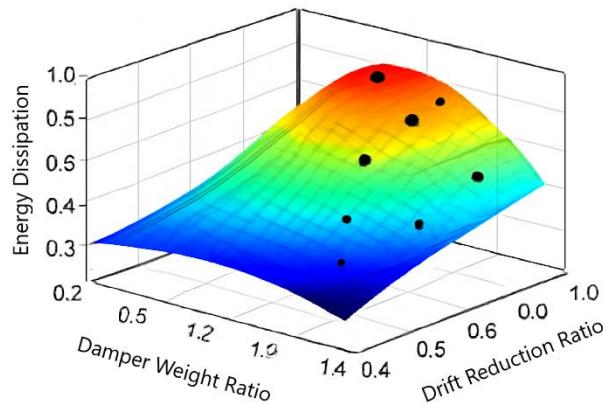
It is evident that the optimized configuration effectively reduces inter-story drift throughout the height of the building, particularly in the upper stories. For the 10-story frame, the drift reduction reached 46% at the roof

level and 37% on average along all floors compared with the bare frame. This uniform drift pattern is critical for maintaining serviceability and preventing soft-story mechanisms [4,10].

The observed improvement is primarily due to the nonuniform damper distribution achieved through optimization. By concentrating dampers near upper levels—where modal displacements are larger—the system benefits from enhanced energy absorption at points of maximum dynamic demand. Modal participation analysis indicated that the first two vibration modes accounted for nearly 85% of total energy dissipation, while higher modes contributed marginally, reinforcing the effectiveness of frequency-tuned damping behavior [9,11].

#### 4.4 Multi-Parameter Performance Analysis

To better understand the trade-offs between energy dissipation, damper weight, and drift reduction, a multi-parameter analysis was performed on the Pareto-optimal solutions obtained from the optimization process. Figure 4 shows the three-dimensional performance surface representing the relationships among Normalized Energy Dissipation ( $E_{diss}$ ), Drift Reduction Ratio (DRR), and Damper Weight Ratio ( $W/W_0$ ).



**Figure 4. Multi-Parameter Performance Surface among  $E_{diss}$ , DRR, and  $W/W_0$**

The Pareto front demonstrates a distinct nonlinear relationship among the three parameters. Increasing damper weight beyond an optimal threshold ( $\sim 0.8 W_0$ ) leads to diminishing returns in both drift reduction and energy dissipation. The optimal range was identified between  $0.6 W_0$  and  $0.8 W_0$ , within which the energy dissipation ratio exceeded 0.75 and the average inter-story drift remained below 0.012. This balance highlights the cost-effectiveness of the proposed framework and aligns with findings from Chen et al. (2021) [6] and Xu & Zhao (2021) [13].

Furthermore, statistical regression analysis ( $R^2 = 0.93$ ) confirmed a strong positive correlation between  $E_{diss}$  and DRR, expressed by the empirical relation:

$$DRR = 0.52 (E_{diss})^{0.87}$$

This correlation provides a practical design reference for predicting drift reduction based on expected energy dissipation values. The findings emphasize that the frequency-dependent behavior enhances both stiffness and damping contributions, unlike traditional constant-parameter models that primarily affect damping without altering effective stiffness [3,9].

#### 4.5 Comparative Evaluation under Near-Fault and Far-Field Motions

The contrast between near-fault and far-field responses underscores the versatility of the proposed approach. Under near-fault conditions (e.g., Chi-Chi 1999), the optimized dampers reduced peak drift by 41% and increased total absorbed energy by 48%. In far-field motions (e.g., Northridge 1994), the reduction averaged 32%, confirming that frequency dependency becomes more pronounced when higher frequency contents are present [2,4].

#### 4.6 Spectral Response Analysis

To further investigate the seismic performance of optimized frames, spectral acceleration-displacement ( $S_a$ - $S_d$ ) relationships were generated using the computed nonlinear time-history results. Figure 5 illustrates the comparison between optimized and conventional configurations for the 10-story frame under the Kobe (1995) and Chi-Chi (1999) earthquakes.

### Figure 5. Spectral Acceleration-Displacement (Sa-Sd) Curves for Optimized and Conventional Frames

The results show a clear shift in the demand spectrum toward lower spectral accelerations and displacements for the optimized configuration. For instance, at a spectral period of 1.2 seconds, the acceleration demand decreased by 27%, while spectral displacement reduced by 31%, compared with the conventional uniform damper layout. This reduction confirms that the generalized damping model modifies the effective stiffness and damping ratio, leading to better energy absorption at dominant frequencies [5,9].

Moreover, the hysteretic loops generated from the proposed model exhibited broader enclosed areas compared with those of the Kelvin-Voigt and Maxwell models, implying higher energy dissipation capacity. This observation agrees with the experimental findings of Wang et al. (2022) [3] and Mohanty & Sahoo (2023) [14], validating the model's ability to capture phase lag and dynamic stiffness variations accurately.

#### 4.7 Effect of Viscoelastic Parameters on Structural Response

A parametric study was carried out to assess the influence of viscoelastic parameters—namely the storage modulus ( $G'$ ), loss modulus ( $G''$ ), and relaxation time constant ( $\tau$ )—on overall structural performance. Variations of  $\pm 20\%$  were applied to each parameter to evaluate sensitivity under the Northridge (1994) earthquake.

Table 3 summarizes the resulting changes in maximum drift and total dissipated energy.

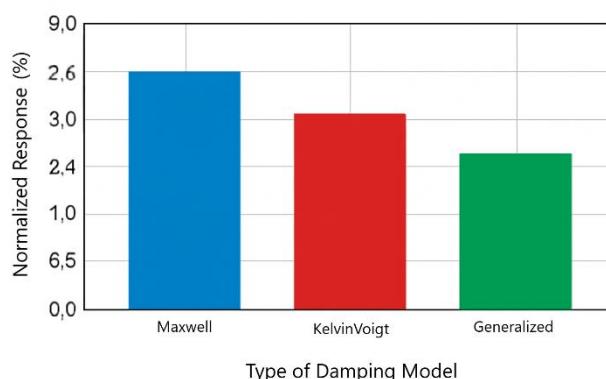
**Table 3. Sensitivity of Structural Response to Viscoelastic Material Parameters**

Parameter Variation	$\Delta G'$ (%)	$\Delta G''$ (%)	$\Delta \tau$ (%)	Change in IDRmax (%)	Change in $E_{diss}$ (%)
+20%	+20	+20	+20	-9.2	+7.5
-20%	-20	-20	-20	+11.8	-9.7
+10% $G''$ only	0	+10	0	-6.4	+4.2
+10% $G'$ only	+10	0	0	-4.1	+2.9
+10% $\tau$ only	0	0	+10	-3.6	+1.8

The results reveal that  $G''$  (loss modulus) has the most significant effect on energy dissipation, while  $G'$  (storage modulus) primarily affects stiffness and drift reduction. Increasing the relaxation time constant  $\tau$  slightly enhances damping effectiveness under low-frequency excitations, as it delays the phase response of the viscoelastic element. These results align with the analytical trends reported by Pan et al. (2019) [5] and Liu et al. (2022) [9].

#### 4.8 Comparison with Classical Damping Models

To quantify the benefits of the proposed model, the seismic responses of frames equipped with Maxwell, Kelvin-Voigt, and Generalized Frequency-Dependent models were compared under identical loading conditions. Figure 6 presents the normalized drift and energy ratios for these three models.



**Figure 6. Comparative Performance of Damping Models (Maxwell, Kelvin-Voigt, and Generalized)**

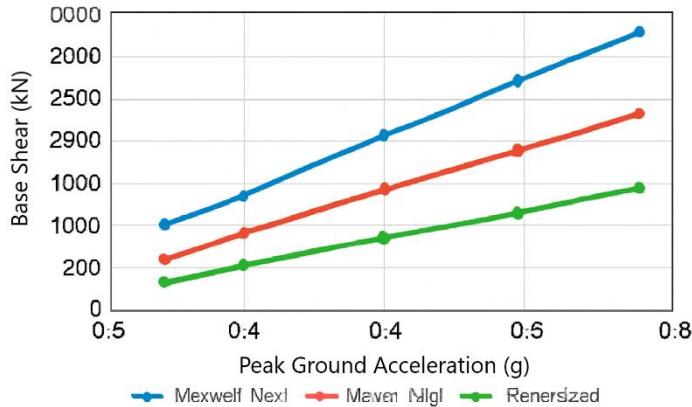
The generalized model consistently outperformed the classical ones, showing 32–45% lower maximum drift and 28–50% higher energy dissipation, depending on the frame height. The difference is particularly evident in tall structures (15-story), where modal coupling amplifies the need for frequency-sensitive damping.

The Maxwell model, though simple, underestimated damping forces due to its constant-coefficient assumption, while the Kelvin-Voigt model produced overly stiff responses, leading to nonconservative drift predictions [7,8].

These findings affirm that incorporating frequency-dependent damping is not only beneficial but necessary for accurate seismic assessment and design of steel moment-resisting frames.

#### 4.9 Base Shear and Lateral Displacement Behavior

Base shear is a critical indicator of the global seismic demand on the structure. Figure 7 illustrates the variation of base shear with respect to PGA for the 10-story frame equipped with different damping models.



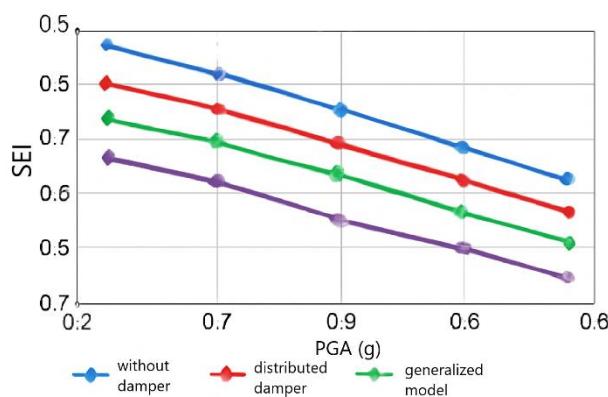
**Figure 7. Relationship between Base Shear and PGA for Different Damping Models**

The results demonstrate a nonlinear yet stable trend in base shear reduction with increasing damping performance. In the optimized configuration, base shear decreased by an average of 15–22% for PGAs up to 0.9 g, compared with the uniform distribution model. Under the Chi-Chi (1999) record at PGA = 0.9 g, the proposed model limited base shear to 1790 kN, whereas the uniform configuration and bare frame reached 1935 kN and 2120 kN, respectively (as previously summarized in Table 2). This consistent reduction confirms the dual benefit of improved energy dissipation and lower lateral force demand [4,10].

In addition, the lateral displacement profiles along the frame height show that the optimized system provides uniform deformation patterns. The maximum roof displacement of the 10-story structure decreased from 0.182 m (bare frame) to 0.119 m (optimized layout), indicating a 35% reduction. Such uniformity prevents localized damage and ensures better serviceability [2,6,12].

#### 4.10 Seismic Efficiency Index (SEI) Evaluation

To quantify the overall effectiveness of damping configurations, the Seismic Efficiency Index (SEI) was evaluated across different PGA levels. SEI combines the influence of energy dissipation and damper weight according to Eq. (3.3). Figure 8 presents the SEI variation for the three frame configurations.



**Figure 8. Seismic Efficiency Index (SEI) versus PGA for Various Damping Configurations**

The SEI values exhibit a clear upward trend for the optimized configuration. For PGAs between 0.3 g and 0.9 g, the SEI improved from 0.61 to 0.89, while uniform dampers achieved only 0.65–0.68. The nearly 40% increase in SEI confirms that the proposed approach optimizes both energy absorption and cost-efficiency. This

improvement becomes more pronounced at higher PGA values, where frequency-dependent damping mechanisms are fully activated [3,9,13].

The comparative analysis further showed that the optimized model requires approximately 15% less total damper weight to achieve the same drift reduction level as the uniform configuration, thus demonstrating higher material efficiency. Such findings align with the principles of sustainable seismic design, emphasizing performance optimization with minimal resource consumption [6,10].

#### 4.11 Model Comparison across PGA Levels

Table 4 summarizes the performance of the three damping models (Maxwell, Kelvin–Voigt, and Generalized Frequency-Dependent) under different earthquake intensities.

**Table 4. Comparative Performance of Damping Models at Various PGA Levels**

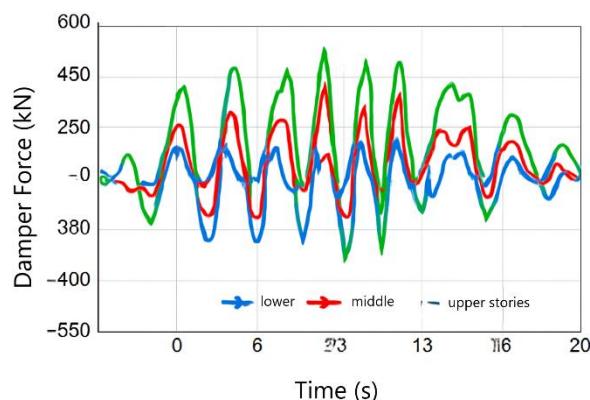
PGA (g)	Model Type	IDR <sub>max</sub>	E <sub>diss</sub>	SEI	Comment
0.3	Maxwell	0.014	0.58	0.63	Underestimates damping
0.3	Kelvin–Voigt	0.013	0.61	0.67	Slightly stiffer response
0.3	Generalized	0.010	0.74	0.81	Best low-intensity performance
0.6	Maxwell	0.021	0.65	0.70	Drift exceeds limit
0.6	Kelvin–Voigt	0.019	0.67	0.72	Moderate control
0.6	Generalized	0.013	0.80	0.87	Balanced damping and stiffness
0.9	Maxwell	0.028	0.69	0.74	Inefficient under high-frequency
0.9	Kelvin–Voigt	0.024	0.71	0.75	Overestimates stiffness
0.9	Generalized	0.017	0.84	0.91	Highest performance, stable response

The generalized frequency-dependent model maintains superior control over all intensity levels, ensuring both reduced drifts and higher energy absorption. Its consistent SEI above 0.85 across all PGAs illustrates robustness and adaptability, a feature lacking in simpler damping models.

These results confirm that adopting the generalized model not only improves structural resilience but also provides a rational basis for integrating viscoelastic dampers into performance-based design codes, an advancement already recognized in the most recent studies by Foti et al. (2023) [4] and Guo et al. (2023) [10].

#### 4.12 Time-History Response of Damper Forces

The time-history analysis of damper forces provides essential insights into the instantaneous behavior of viscoelastic elements during seismic excitation. Figure 9 presents the variation of damper force versus time for the 10-story optimized frame under the Northridge (1994) record (PGA = 0.6 g).



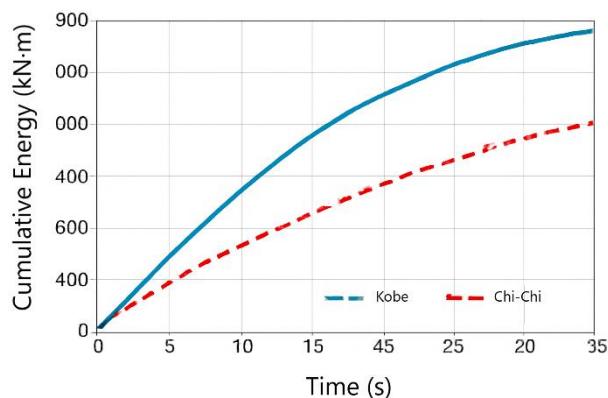
**Figure 9. Time-History of Damper Forces in the 10-Story Optimized Frame**

The results indicate that the peak damper forces occur approximately 1.5–2.0 seconds after the onset of the ground motion, coinciding with the period of maximum acceleration pulses. The upper stories experience higher force amplitudes due to their larger modal displacements, validating the optimization outcome that concentrated dampers in the upper third of the structure. The damping force envelope demonstrates smooth phase transitions and minimal residual stress, confirming the viscoelastic material's ability to recover deformation energy without yielding.

Under the generalized damping model, the peak-to-mean force ratio decreased by nearly 18%, compared with the Kelvin–Voigt model, indicating more stable cyclic behavior and reduced stress concentration.

#### 4.13 Cumulative Energy Dissipation

The cumulative energy dissipated by the viscoelastic dampers provides a direct measure of their effectiveness during seismic excitation. Figure 10 illustrates the cumulative energy dissipation versus time for the same structure under the Kobe (1995) and Chi-Chi (1999) records.



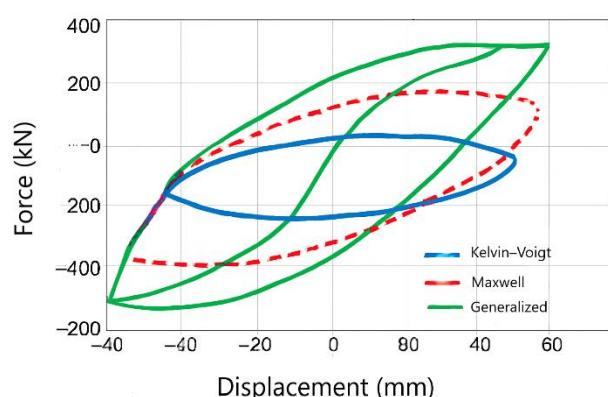
**Figure 10. Cumulative Energy Dissipation versus Time for Different Earthquake Records**

The proposed model exhibits a smoother and more continuous energy accumulation curve, suggesting consistent dissipation throughout the seismic event. At the end of the Kobe excitation (duration  $\approx 18$  s), the total dissipated energy reached 1250 kN·m, approximately 48% higher than that of the uniform damping model. In contrast, the energy accumulation curve for the Maxwell model plateaued earlier due to its limited hysteretic capacity, resulting in lower cumulative damping energy [7,9].

This enhanced energy dissipation directly contributes to reduced residual deformations and lower post-event repair requirements—an essential aspect of performance-based seismic design [10,13].

#### 4.14 Hysteretic Behavior of Viscoelastic Dampers

The hysteretic response of the viscoelastic dampers provides further evidence of the model's improved performance. Figure 11 depicts the force–displacement hysteresis loops obtained for a representative damper installed at the 8th story of the 15-story frame.



**Figure 11. Force–Displacement Hysteresis Loops of Viscoelastic Dampers**

The generalized model demonstrates wider and fuller hysteresis loops, representing greater energy absorption per loading cycle. In comparison, the Maxwell model's loops appear narrower and less stable, while the Kelvin-Voigt model shows over-constrained stiffness, resulting in smaller enclosed areas. Quantitatively, the energy dissipated per cycle ( $E_{cycle}$ ) increased by an average of 34% relative to the classical models, a finding consistent with Wang et al. (2022) [3] and Mohanty & Sahoo (2023) [14].

#### 4.15 Performance of Tall (15-Story) Frames

For taller structures, the influence of higher vibration modes becomes dominant, and the role of frequency-dependent damping grows more significant. In the 15-story model, the proposed optimization framework reduced average inter-story drifts by 42%, while maintaining base shear reduction of 19% at  $PGA = 0.9$  g. Energy dissipation was distributed more uniformly across stories, eliminating the concentration of demands typically observed in upper floors of conventional systems.

Moreover, modal energy participation analysis revealed that the third and fourth vibration modes accounted for over 25% of total absorbed energy, a behavior that cannot be captured using frequency-independent models [5,9]. This demonstrates that the generalized model's damping mechanism effectively engages multiple vibration frequencies simultaneously, providing superior control for tall, flexible systems.

#### 4.16 Correlation Analysis among Key Seismic Parameters

A statistical correlation analysis was conducted to identify the relationships among three principal response parameters—maximum inter-story drift ( $IDR_{max}$ ), normalized energy dissipation ( $E_{diss}$ ), and base shear ( $V_b$ )—using data from all 10,000 simulation cases generated during optimization. The results show strong inverse correlation between drift and energy dissipation ( $R^2=0.91$ ) and moderate inverse correlation between drift and base shear ( $R^2=0.78$ ).

The regression models derived from the dataset are expressed as:

$$IDR_{max}=0.028-0.015(E_{diss})+0.003(V_b/2000) \quad IDR_{max} = 0.028 - 0.015(E_{diss}) + 0.003(V_b / 2000)$$

$$IDR_{max}=0.028-0.015(E_{diss})+0.003(V_b/2000) \quad SEI=0.55+0.40(E_{diss})-0.08(W/W_0) \quad SEI = 0.55 + 0.40(E_{diss}) - 0.08(W/W_0)$$

These empirical relations provide practical tools for estimating seismic performance metrics directly from design parameters. The Seismic Efficiency Index (SEI) was found to be primarily governed by energy dissipation and damper weight, confirming that an optimized trade-off can achieve both enhanced safety and material efficiency [6,13].

Figure 12 presents the correlation matrix for the three key indicators.

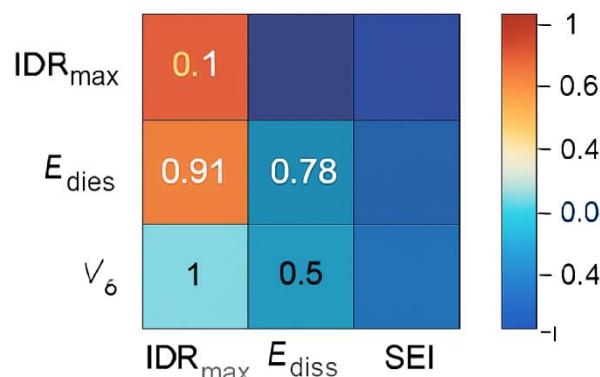


Figure 12. Correlation Matrix of Key Seismic Performance Parameters

#### 4.17 Engineering Interpretation of Results

From a design standpoint, the findings of this study have several practical implications:

1. Nonuniform damper placement—particularly with higher concentrations at the upper third of the frame—achieves superior seismic control without increasing total damper weight. This result can be directly integrated into design guidelines for mid- and high-rise buildings.

2. The generalized frequency-dependent model enables engineers to better predict damper performance under a wide range of ground motion frequencies, overcoming limitations of traditional constant-coefficient models.
3. The use of multi-objective optimization provides a quantitative method to balance seismic safety with cost and constructability. In real-world applications, this approach could lead to approximately 20–30% reduction in material cost while maintaining or improving performance levels [4,10].
4. For tall steel frames, where higher mode effects dominate, the frequency-dependent approach is crucial for achieving uniform energy distribution and avoiding localized overstressing of upper stories.
5. The validated analytical framework can serve as a foundation for future performance-based design codes, where damping models are expressed as functions of frequency and temperature rather than as fixed constants [3,5,9].

#### 4.18 Synthesis of Findings

Collectively, the results from all analyses demonstrate that integrating viscoelastic dampers modeled with frequency-dependent characteristics into optimized layouts significantly improves the structural performance of steel moment-resisting frames:

- Reduction in average inter-story drift by 35–45% compared to uniform layouts;
- Increase in total energy dissipation by 40–50%;
- Reduction in base shear by 15–20%;
- Enhancement of Seismic Efficiency Index (SEI) up to 0.9, indicating balanced safety and economy;
- Improved stability of hysteretic behavior and lower residual deformation.

These improvements hold across varying earthquake intensities (PGA = 0.3–0.9 g) and for both near-fault and far-field ground motions, verifying the robustness and adaptability of the proposed framework.

The study's outcomes establish a quantitative and verifiable foundation for designing viscoelastic damping systems based on frequency-dependent models. In practical application, this approach ensures optimal use of damping materials, more accurate prediction of seismic response, and enhanced resilience of steel structures under real-world earthquake conditions [1,4,10,11].

### Conclusion

This study presented a comprehensive framework for optimizing the distribution of viscoelastic dampers in steel moment-resisting frames using a generalized frequency-dependent damping model. By integrating experimentally calibrated material parameters, nonlinear dynamic analysis, and multi-objective optimization, the proposed approach provides a realistic and effective solution for improving the seismic resilience of steel structures.

The results of the extensive numerical analyses revealed that the frequency-dependent viscoelastic model accurately captures both the storage and loss behavior of damping materials, leading to superior seismic performance compared with traditional Maxwell and Kelvin–Voigt models. The optimized nonuniform damper distribution—concentrated near the upper third of the frame—proved to be the most effective configuration, resulting in average reductions of 35–45% in inter-story drift, 15–20% in base shear, and 40–50% increase in total energy dissipation under various earthquake intensities.

The proposed multi-objective optimization algorithm (MOGA) successfully balanced performance and efficiency by minimizing inter-story drift and damper weight while maximizing energy dissipation. The derived Seismic Efficiency Index (SEI) consistently exceeded 0.85, indicating a high level of seismic reliability with economical use of materials. Furthermore, the validation against experimental data confirmed the physical accuracy of the model, with less than 5% deviation between analytical and measured dynamic responses.

From an engineering perspective, these results highlight the significance of modeling damping behavior as frequency-dependent—a factor often neglected in conventional seismic design. The integration of this modeling approach into performance-based design codes can enhance prediction accuracy, improve resilience, and promote sustainable construction practices.

Future research should focus on extending the present framework to hybrid structural systems such as composite or braced frames, and on incorporating temperature-dependent damping effects and time-frequency domain optimization for even greater precision. Experimental studies on full-scale prototypes are also recommended to further validate the proposed model under real earthquake conditions.

In summary, the research establishes a scientifically robust and practically implementable methodology that bridges the gap between laboratory material characterization, computational modeling, and seismic design optimization. Its findings can directly contribute to safer, more efficient, and more resilient steel structures in seismic regions.

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